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# The NP-Completeness Column: An Ongoing Guide

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This is the tenth edition of a quarterly column that provides continuing coverage of new developments in the theory of NP-completeness. The presentation is modeled on that used by M. R. Garey and myself in our book ''Computers and Intractability: A Guide to the Theory of NP-Completeness,'' W. H. Freeman & Co., New York, 1979 (hereinafter referred to as ''[G&J]''; previous columns will be referred to by their dates). A background equivalent to that provided by [G&J] is assumed, and, when appropriate, cross-references will be given to that book and the list of problems (NP-complete and harder) presented there. Readers who have results they would like mentioned (NP-hardness, PSPACE-hardness, polynomial-time-solvability, etc.), or open problems they would like publicized, should send them to David S. Johnson, Room 2C-355, AT&T Bell Laboratories, Murray Hill, NJ 07974, including details, or at least sketches, of any new proofs (full papers are preferred). If the results are unpublished, please state explicitly that you would like them to be mentioned in the column. Comments and corrections are also welcome. For more details on the nature of the column and the form of desired submissions, see the December 1981 issue of this Journal.

## 1. INTRODUCTION

It is now two and a half years since I began this column, and the backlog of uncited results is substantially larger than when I started. This month, in hopes of making a sizable dent in this backlog, I will be presenting a large number of results on a variety of topics. The common strain linking these results is that they are all, in a sense, embroidery on previously presented results. That is, they are complexity results for special cases of, or close variants on, problems already reported to be NP-hard in earlier editions of this column (corrections to my own imprecisions will also be included, as well as one author's retraction).

In order to include as many results as possible, I will not in general repeat the full definition of the original problem, but will only give its name, the location of its definition, and perhaps a few words of informal description to jog the reader's memory. The resulting telegraphic presentation may seem a bit cryptic to readers not familiar with the original problems. However, those not familiar with the original problems will probably not be all that interested in further details anyway. A good reading plan would be to skip over those entries that seem irrelevant to your current interests, and go directly to those (if any) that catch your attention. The updates, with one exception, are presented in Section 2, ordered chronologically by the columns to which they refer. The exception is a single problem that, with almost 1/4 of this month's references all to itself, clearly deserves the Embroidery Championship of the World, and thus gets the concluding section all to itself.

The 86 references cited this month exemplify the sheer bulk of results related to NP-completeness that are currently being generated. The column cannot, however, take the credit (or blame) for much of this outpouring, since many of the ''updates'' were obtained independently of (and often previously to) the results on which they elaborate. I still find as many results in the library as I do in the mail (and just recently I unearthed a box full of relevant results that I had put into storage just before going on sabbatical in 1980; this is the source of several ''backdates'').

Since I am admitting my shortcomings as a result collector, I might as well own up to another area of slippage, especially relevant this month when I must retract one previously mentioned result and correct several others. The set of ''trusted colleagues'' who were going to help me ''verify all NP-hardness results presented'' [Dec. 1981] has gradually expanded beyond my co-workers and co-authors to include thesis supervisors, anonymous journal referees, and, as the bulk has grown (and when the result is what I would expect) even such riff-raff as the authors themselves. (The riff-raff who ''verified'' the proof of the retracted result was, however, yours truly, so it is not clear that the standards have really been lowered.) Readers should thus keep in mind that this ''ongoing guide'' *is* just a guide; for results you are particularly interested in, there is no substitute for the primary sources.

## 2. UPDATES, UPDATES, UPDATES

In this section I will quickly traverse columns 1 through 9, providing corrections and brief updates. I will have something to say about all the columns except Number 6 [March 1983], whose discussion of linear programming and knapsack cracking will be updated in the near future.

**CHROMATIC INDEX** [Dec. 1981]. Remains NP-complete even if the chromatic index of the *complement* of *G* equals the obvious lower bound of maximum vertex degree [18]. Solvable in polynomial time for outerplanar graphs [65] and for planar graphs with a vertex of degree 8 or more (in which case the chromatic index equals the maximum vertex degree [81,83]). The polynomial time solvable case of bipartite graphs becomes NP-complete when generalized by adding to the instance a partition of the edges and requiring that no two edges in the same subset of the partition be assigned the same color [8], a result that holds even if the maximum vertex degree and set cardinality are both 3 (if they are both 2, polynomial time solvability returns).

**HAMILTONIAN CIRCUIT** [GT37] and [Dec. 1982]. I will not repeat the developments already reported in the [Dec. 1981], [March 1982], and [Sept. 1982] columns, except to note that I've discovered an earlier proof of the result that HAMILTONIAN PATH can be solved in polynomial time for reducible flow graphs [34], and that an independent proof of NP-completeness for edge-graphs has appeared [5], thus supplanting the previously cited ''unpublished result'' [Dec. 1981]. Here are some new, or at least previously unmentioned, polynomial time solvable subcases: series-parallel graphs [69], proper interval graphs (interval graphs corresponding to collections of intervals, no one of which is completely contained in another) [7], proper circular arc graphs that are not proper interval graphs [7], 4-connected planar graphs [31], and graphs obeying a degree-sequence characterization of Chvátal [10,17]. (The last three results are slight cheats, however, since for these classes it was already known, or is easy to see, that the graphs in question *always* have Hamiltonian circuits. Thus the trivial algorithm that always says ''yes'' solves HAMILTONIAN CIRCUIT for these graphs; the papers provide algorithms for the different task of *constructing* the circuit.)

**MINIMUM CUT LINEAR ARRANGEMENT** [GT44] and [March 1982]. Can now be solved in polynomial time for arbitrary trees [86], not just the degree-restricted trees of previous results.

**WEIGHTED GRAPH EMBEDDABILITY** [March 1982]. This problem asks whether we can embed an edge-weighted graph into $R^K$ in such a way that the Euclidean length of each edge equals its weight. The complexity of the $K = 1$ case was misstated in the cited column. This case is in P if all weights are equal and is NP-complete if weights come from the set $\{1,2\}$ [74]. A new result concerns the variant in which $K = 2$ and the embedding must be planar, i.e., no edges can intersect except at a common endpoint. This problem is NP-hard for planar 2-connected graphs even if all weights are equal [23]. A related new result concerns the NP-complete variant in which all vertices must map to distinct *integer coordinate* points in the plane. This remains NP-complete even if all weights (i.e., edge lengths) are 1, and even if $G$ is a tree [9]. This is a particularly nice result because it implies the two special case results that I claimed I could prove in the [March 1983] column, and thus frees me from the need to provide the details of my proofs. More significantly, see the next entry and the comments below on BANDWIDTH [GT40].

**MINIMUM AREA EMBEDDING OF PLANAR GRAPHS** [March 1982]. The previously reported result was for embeddings where the paths representing edges are not allowed to cross each other, and holds for not-necessarily-connected forests. The problem is now known to be NP-complete if paths are allowed to cross, even if $G$ is connected (but not necessarily planar) [44]. The variant which asks for the grid embedding of a graph that minimizes maximum *edge length* is no longer open. NP-completeness holds for both the case where the paths representing distinct edges may not intersect and the case where they may, but only at grid points. This follows from the above-mentioned result of [9]. If the grid into which the graph is to be embedded is defective (certain grid segments and points are unusable), then the problem of determining whether *any* embedding is possible becomes relevant, but is NP-complete even for paths, no matter whether crossings are allowed or not, due to the NP-completeness of HAMILTONIAN CIRCUIT for grid graphs [37]. If crossings are not allowed, it is also NP-complete for binary trees (trees in which all vertices have degree 1 or 3) [82].

**WEIGHTED TREE LAYOUT WITH FIXED LEAVES** [March 1982]. This problem was known to be solvable in polynomial time if edges must be represented by rectilinear paths and pointwise intersections are allowed. If the rectilinear paths are allowed to intersect only at common endpoints, the problem becomes NP-complete [3].

**CROSSING NUMBER** [March 1982]. Also NP-complete if we require that the embedding into the plane be a grid embedding [75].

**PARTITION INTO TRIANGLES** [GT11] and [June 1982]. NP-complete for planar graphs [4,22], and hence the problem of finding a collection of vertex-disjoint triangles that covers a maximum number of vertices in a planar graph $G$ is NP-hard. However, if one wants a collection of vertex-disjoint $K_3$'s and $K_2$'s, rather than just $K_3$'s, this latter problem can be solved in polynomial time, even for non-planar graphs [19].

**EDGE-PARTITION INTO TRIANGLES** [June 1982]. Remains NP-complete even if there *is* an edge-partition into triangles for the complement of $G$ [18].

**DISTANCE-d CHROMATIC NUMBER** [June 1982]. This problem was motivated by the problem of approximating sparse Hessians. NP-completeness results for other coloring problems arising from this application can be found in [56].

**DISTANCE-d PARTITION OF POINTS IN THE PLANE** [June 1982]. A recent archaeological expedition into my pre-1981 files has unearthed a private communication indicating that this result should also be attributed to J. Orlin [61].

**GEOMETRIC COVERING BY DISCS** [June 1982]. Another archaeological find: the NP-completeness of the version of this problem where discs are replaced by squares should also be credited to D. Kirkpatrick [43], as should the NP-completeness of the corresponding packing problem (given a set of points in a rectangle $R$, can $k$ disjoint unit squares that avoid all the given points be placed inside $R$?).

**CUT INTO CONNECTED COMPONENTS OF BOUNDED SIZE** [June 1982]. Not only is this problem (trivially) solvable in polynomial time for trees; it is also solvable in polynomial time if $G$ is series-parallel, so long as the two components are required to be of equal size (see [21] for this and related results).

**PARTITION INTO CONVEX REGIONS** [June 1982]. The polynomial time algorithm of [16] for partitioning a hole-free polygon $Q$ into a minimum number of convex regions still has not, to my knowledge, been verified. However, if one is willing to forego the use of Steiner points, a polynomial time algorithm for this problem is available in [40], which also has polynomial time algorithms for the variants in which ‘‘convex’’ is replaced by ‘‘spiral,’’ ‘‘star-shaped,’’ or ‘‘monotone.’’ Introducing holes into $Q$ makes all four variants of the Steiner-pointless problem NP-hard [40] (the original proofs in [51] required Steiner points). Analogous algorithms exist for minimizing the sum of the region perimeters rather than the number of regions, with the introduction of holes once again causing NP-hardness, at least in the ‘‘convex’’ variant [40]. (This variant is also NP-hard if Steiner points are allowed [50].) The other variants remain open.

**PACKING WITH SQUARES** [June 1982]. Remains NP-complete if the $2\times2$ square is replaced by *any* rectangle of integer length and width except for the trivial $1\times2$ and $1\times1$ cases [4]

**BIN PACKING** [SR1] and [Sept. 1982]. The final version of the Karmarkar-Karp approximation algorithm, described in [38], provides an even better worst-case guarantee than I had estimated based on a preliminary draft. In polynomial time, it guarantees a number of bins that is no more than $\mathrm{OPT}(I) + O(\log^2 \mathrm{OPT}(I))$.

**BANDWIDTH** [GT40] and [Sept. 1982]. Although, as previously reported, this problem can be solved in polynomial time for caterpillars with hair length at most 2, letting one’s hair grow has the usual dire consequences: NP-completeness sets in as soon as hairs of length 3 are allowed [58]. The generalization in which one wishes to minimize the maximum edge length when the vertices of $G$ are assigned to distinct integer coordinate points in $R^2$ rather than $R^1$ (2-DIMENSIONAL BANDWIDTH) is NP-hard even if we ask about bandwidth 1, as follows from the abovementioned result about weighted graph embeddability in the grid [9] (the NP-completeness of unrestricted 2-DIMENSIONAL BANDWIDTH was first shown in [57]).

**SHUFFLED STRING** [Sept. 1982]. If there are only $k$ strings to be shuffled, for some fixed $k$, the problem can be solved in polynomial time [54,55]. (Reference [55] also proves the problem NP-complete if $k$ is not fixed, but this result is not as strong as the one I originally reported [84], which holds for alphabets of size 2.)

**STEINER TREE IN GRAPHS** [ND12] and [Dec. 1982]. Although NP-complete for planar graphs $G$, the problem can be solved in polynomial time for such graphs if the set $R$ of points to be connected is all on the same face in a planar embedding of $G$, even if arbitrary edge weights are allowed [24]. This can be generalized to the case where the points are all on some set of $K$ faces, for $K$ fixed [66]. It also can be applied to the rectilinear version of GEOMETRIC STEINER TREE [ND13] to obtain an algorithm for the case where all the points to be connected are on the boundary of a ‘‘convex’’ rectilinear polygon [66], thus extending the results of [1]. A related result concerns the directed version of the problem, in which we are given an edge-weighted directed graph $G$, a specified source vertex $s$, and a set $T$ of sink vertices, and are asked for a subgraph of minimum weight that contains a directed path from $s$ to $t$ for every $t \in T$. This variant can also be solved in polynomial time for planar graphs, so long as all the vertices in $T$ lie on a single face [66]. Without this restriction, however, it is NP-complete even for acyclic planar graphs with all edge weights equal [66].

**BUS ROUTING** [Dec. 1982]. This problem is actually NP-complete in the strong sense [73], and the parenthetical remark I made about the polynomial time solvability of the not-necessarily-acyclic directed graph variant should of course have included the proviso that the ‘‘arbitrary’’ weights not be so arbitrary as to be negative.

**PLANAR INTERCONNECTION ON A GRID** [Dec. 1982]. Additional credit for the main result should go to [67]. Also NP-complete even if each interconnecting path must be ‘‘monotone’’ in the horizontal direction (must proceed from left to right without turning back) [39]. If, instead of asking just whether a routing is possible, we ask for a routing of minimum total wire length, given that arbitrary non-intersecting paths are allowed, we still have NP-hardness [49]. NP-hardness also holds if ‘‘non-intersecting’’ is replaced by ‘‘at least distance $D$ apart,’’ and is conjectured to hold if only paths made up of horizontal and vertical line segments are allowed [49].

**TWO-LAYER CHANNEL ROUTING** [Dec. 1982]. If only two-terminal nets are allowed, one can tell in polynomial time whether there is an edge-disjoint routing (the paths joining the different pairs intersect only at grid points) that uses a given number of ‘‘tracks’’ [28]. This is almost a polynomial time solution to the two layer ‘‘knock-knee’’ channel routing problem, but not quite. In a true knock-knee routing, paths are allowed to intersect at grid points, but account is taken of the fact that paths must be in different layers if they intersect, and when paths change layers they must do so at grid points where they do *not* intersect other paths [63]. Merely because an ensemble of paths is edge-disjoint does not guarantee that a consistent assignment of path segments to layers can be made subject to the above constraints. However, now consider the problem of *three* layer knock-knee routing, where a path can change layers at a grid point where it intersects a second path, so long as the two paths don't share a layer at that point and the second path does not occupy the middle layer. Here one can not only find an optimal routing in polynomial time, but can in fact always attain the obvious ''density'' lower bound on channel width [63]. Moreover, here the distinction between edge-disjoint routings and knock-knee routings can be illustrated by a complexity result. The following problem is NP-complete: given an ensemble of edge-disjoint paths connecting the (2-terminal) nets in a channel routing instance, is there a way of assigning layers to path segments so that a valid 3-layer knock-knee routing is obtained? [53].

For results concerning variants in which one attempts to improve routability by altering the spacing between the terminals along the top and the bottom of the channel, see [30].

**MINIMUM LENGTH PSEUDO-TRIANGULATION** [Dec. 1982]. This is the problem whose NP-completeness proof has been withdrawn. More precisely, in the final version [52] of the original ''unpublished manuscript,'' the result is only claimed to hold if at least one of a pair of quite plausible (and quite technical) conjectures is true.

**PRECEDENCE CONSTRAINED SCHEDULING** [SS9] and [June 1983]. The 2-processor problem with arbitrary precedence constraints and task lengths 1 and 2, long known to be NP-complete, is solvable in polynomial time if only tree-structured precedence constraints are allowed, although generalizations to lengths 1 and $k$ or to more than 2 processors have not yet been obtained [59].

**DEADLOCK POTENTIAL** [Sept. 1983]. The following variant is solvable in polynomial time. There are $n$ process types $P_1, \ldots, P_n$, each process of type $i$ never needing more than a given number $C(i,j)$ of units of resource $R_j$ $1 \le j \le m$, but always capable of requesting extra units when it is below that bound. At any time the system can contain a maximum of $K$ processes. Given this information and the resource bounds, one can in polynomial time determine whether deadlock is possible using a degree-constrained matching algorithm [36]. However, the problem becomes NP-hard if we place individual bounds on the allowed numbers of each type of process, or if we try to find a set of resource bounds that guarantees freedom from deadlock while minimizing a linear cost function [36].

**DEADLOCK RECOVERY** [Sept. 1983]. This update involves more recovery from error than recovery from deadlock. In the results of [47,48], one is actually looking for a minimum cost set $P'$ of processes such that, if they are all aborted and their resources returned to the common pool, then there will be a way for the remaining processes to proceed to completion in some *sequence* (not necessarily all simultaneously). In other words, some process will be able to take all the resources it is requesting, after which it will eventually finish, returning all the resources it holds and thus enabling another process to proceed. (There is, of course, no guarantee that the processes will be so patient and well-behaved, but at least they *could* avoid future deadlock if they wanted to).

**UNSAFE LOCKING POLICY** [Sept. 1983]. The distributed two-transaction version of this problem, known to be NP-complete for an arbitrary number of sites but polynomial when restricted to just two sites, continues to be in P when the number of sites is upped to 3 [80]. It remains open for fixed numbers of sites exceeding 3.

**DISTRIBUTED SERIALIZABILITY ASSURANCE** [Sept. 1983]. A new NP-completeness result has been proved concerning the related problem of minimizing the number of database operations that must be ''backed out'' in order to maintain the overall serializability of a system of transactions in a distributed system where duplicate copies of data are held at separate sites. For details (and independent proofs), see [20,85].

**THE PEBBLE GAME** [Dec. 1983]. The following ''single pebbling'' variant is NP-complete: The vertices of $G$ are partitioned into ''type'' classes, a vertex may only be pebbled by a pebble of its type, and there is exactly one pebble of each type. Subject to these added constraints, is there a pebbling strategy that pebbles each vertex of $G$ exactly once? This variant is motivated by storage allocation problems related to the handling of global attributes in an attribute grammar and to the serializability of database updates. Its NP-completeness proof is constructed by putting together an observation in [77] with a construction in [76]. See [68] for more involved problems and related results.

**N × N GO** [GP11] and [Dec. 1983]. As predicted, this problem has been proved complete for EXPTIME [70,71]. My informant also claims that, if one adopts the Chinese Go rule that forbids repetition of position, the game becomes complete for EXPSPACE [72].

## 3. THE EMBROIDERY CHAMPION OF THE WORLD

**DOMINATING SET** [GT2] and [June 1982]. Recall that this problem asks for a minimum-sized subset $V'$ of the vertices such that every other vertex is adjacent to some member of $V'$. The obvious application for this problem and its variants is to the optimum location of facilities in a network, but it apparently also comes up in questions relating to coding theory [79].

For some reason, there has recently been a remarkable outpouring of results related to this problem, and indeed, one of the most-requested proofs of an ''unpublished result'' by the authors of [G&J] concerns it. The requests concern the report in [G&J] that this problem and its variant CONNECTED DOMINATING SET (in which $V'$ is required to induce a connected subgraph) are both NP-complete for planar graphs that are regular of degree 4. These results, unfortunately, seem to have been the product of an error in transcription rather than a polynomial transformation. According to my notes, the classes for which NP-completeness was proved in [29] are, for DOMINATING SET, 3-regular planar graphs (proved independently in [42]), and for CONNECTED DOMINATING SET, the two classes (a) 4-regular (not necessarily planar) graphs and (b) planar graphs with maximum degree 4. The *new* developments in domination come from all over the world and are summarized in the following paragraphs.

Unlike the closely related VERTEX COVER problem, DOMINATING SET is NP-complete for bipartite graphs [6,14] and for chordal graphs [6,12,14]. The latter result was already mentioned in the [June 1982] column, as a corollary of NP-completeness for ''undirected path graphs'' [12]. It is now also a corollary of the NP-completeness of a different subclass of chordal graphs: ''split graphs,'' i.e., those graphs $G = (V,E)$ whose vertex sets $V$ can be partitioned into two sets $V_1$ and $V_2$, such that the subgraph induced by $V_1$ is a clique and the one induced by $V_2$ is an independent set. The results for bipartite and split graphs can be extended to CONNECTED DOMINATING SET, TOTAL DOMINATING SET (in which each vertex, *including those in* $V'$, must be adjacent to some member of $V'$), and IRREDUNDANCE NUMBER [45,62]. (The *irredundance number* of a graph is the minimum size of a maximal irredundant set, where $v \in V'$ is *redundant* if it is adjacent to another member of $V'$ and each of its neighbors is either in $V'$, or adjacent to some vertex in $V' - \{v\}$; note that the irredundance number is no larger than the size of a minimum dominating set, and can be smaller [45].) The undirected path graph result has been extended to TOTAL DOMINATION [46]. However, the result for split graphs cannot be extended to INDEPENDENT DOMINATING SET (in which the dominating set must also be an independent set), since this problem can be solved in polynomial time for chordal graphs, even if weights of 0 and 1 are allowed on the vertices [26] (although it too is NP-complete for general graphs, and even for chordal graphs if arbitrary weights are allowed [13]).

Ordinary DOMINATING SET can be solved in polynomial time for ''strongly chordal'' graphs [27], circular arc graphs [11], cacti [33], series-parallel graphs [41], and for graphs that differ from trees by only a fixed number of edges [32]. It is easy to see that all the above DOMINATING SET variants can be solved in polynomial time for trees (even in weighted versions, e.g., see [60]). The only current exception is IRREDUNDANCE NUMBER, which remains open in this case. I'm not sure whether the polynomial time algorithm for DOMINATING SET on ''directed path graphs'' of [12] can be extended to the other variants (nor am I sure that I really want to know). Also solvable in polynomial time for trees with weighted edges is the $k$-DOMINATING SET problem, where every vertex must be within distance $k$ of some vertex in the dominating set [35], and the $R$-DOMINATING SET problem, which is a generalization of this to allow required distance limits to vary from vertex to vertex (among other things) [78]. $R$-DOMINATING SET can also be solved in polynomial time for the generalization of trees to ''block graphs'' [15].

Yet another variant is the one in which the dominating set is required to induce a cycle. This version is NP-complete for planar 2-connected graphs, but solvable in polynomial time for *outer*planar graphs [64]. Finally, consider DIRECTED DOMINATING SET, in which one asks for a minimum-sized subset $V' \subseteq V$ such that for each $u \in V - V'$ there exists an arc $(v,u)$ for some $v \in V'$. The NP-completeness of this problem follows from that for the undirected version, but here, under certain restrictions, the mere *existence* of a dominating set is difficult to determine. In particular, for any fixed $k \geq 2$, it is NP-complete to determine whether there exists a dominating set whose induced graph contains no path of $k$ or more arcs [2]. (As this month's last kernel of wisdom, I note that, for $k = 1$, the question of whether such a dominating set exists is just the NP-complete KERNEL problem [GT57].)

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