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The NP-Completeness Column: An Ongoing Guide

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This is the twentieth edition of a (usually) quarterly column that covers new developments in the theory of NP-completeness. The presentation is modeled on that used by M. R. Garey and myself in our book ''Computers and Intractability: A Guide to the Theory of NP-Completeness,'' W. H. Freeman & Co., New York, 1979 (hereinafter referred to as ''[G&J]''; previous columns will be referred to by their dates). A background equivalent to that provided by [G&J] is assumed, and, when appropriate, cross-references will be given to that book and the list of problems (NP-complete and harder) presented there. Readers who have results they would like mentioned (NP-hardness, PSPACE-hardness, polynomial-time-solvability, etc.) or open problems they would like publicized, should send them to David S. Johnson, Room 2D-150, AT&T Bell Laboratories, Murray Hill, NJ 07974 (or to dsj%btl.csnet@relay.cs.net). Please include details, or at least sketches, of any new proofs; full papers are preferred. If the results are unpublished, please state explicitly that you are willing for them to be mentioned. Comments and corrections are also welcome. For more details, see the December 1981 issue of this Journal.

ANNOUNCEMENTS, UPDATES, AND GREATEST HITS

This is the twentieth edition of the NP-Completeness Column, and as with the tenth will be devoted to updates on previous columns and various housekeeping matters. It will also be relatively short. The recently established pattern, in which the Nth column has been N pages long, has been as exhausting to the author as it has no doubt been to his readers.

I begin in Section 1 with an announcement, inspired in part by the recent flurry of claimed resolutions to the question of P versus NP. Section 2 is then devoted to updates and correction to recent columns. (The previously announced book collecting and updating the first fourteen columns is still in the works for Academic Press, although the planned publication date has now slipped at least into early 1988.) Section 3 concludes with a first for the NP-Completeness column: an actual NP-completeness proof (one of the two most requested unpublished results from [G&J]).

1. An Announcement

Anyone tuned into the wavelength for theoretical computer science gossip knows that, over the last year or so, there has been a veritable eruption of purported proofs that P = NP. It is not clear whether this is a coincidence or a communicable disease, but it was clearly presaged by a short item that appeared in the program for the April 1986 TIMS/ORSA Joint National Meeting in Los Angeles. On page 223 of the program was a listing for a talk (WD.08) by one ''M. Twain'' of the University of Missouri. The talk was entitled ''On the equivalence of two complexity classes,'' and its one-line abstract read, ''We use the theory of recursive functions to show that the P = NP conjecture is a mother functor thereby settling the conjecture.''

Needless to say, Mr. Twain never showed up to deliver his speech. In the year and a half that has followed, however, at least six *non*-pranksters have claimed resolutions to the P versus NP question, including E. R. Swart of the University of Guelph and M. A. H. Dempster of the University of Dalhousie, both of whose claims have been widely publicized and discussed, as well as several more circumspect researchers. Swart was first, with a widely-circulated technical report entitled ''P = NP'' that claimed a polynomial-time algorithm for HAMILTONIAN CIRCUIT based on linear programming [37]. More recently, Dempster used the same title for an abstract in *this* year's TIMS/ORSA conference program (talk [MB19], p. 29), and claimed to be able to solve CLIQUE in polynomial time using nonlinear programming methods.

Unfortunately (and as expected) none of the six claims has survived critical examination. Given the significance of the question they addressed, however, it seems a shame that they should all have died in vain. Therefore, Michael Garey and I announce the establishment of the MEMORIAL REPOSITORY FOR PROOFS THAT P = NP (AND ITS NEGATION). Those having proposed resolutions to the P versus NP question are urged to send a copy to me at the address given at the beginning of the column (and those finding holes in the above are likewise urged to report). Proofs that the question is independent of set theory are also welcome.

For historical purposes, we are in addition interested in collecting information about retracted claims from the past. (Our favorite is a supposedly polynomial-time algorithm for SATISFIABILITY that made use of a new logic system based on the Old Testament Book of Genesis.) We will of course honor requests for anonymity, as our main goal is to gather information on the techniques used and approaches attempted, and to look for trends. (Currently it seems that proofs that P = NP outnumber proofs to the contrary by a ratio of over 5 to 1.)

The serious technical purpose in all this lies in the fact that even false proofs may occasionally yield new and useful insights, as can the exercise of finding counterexamples. For instance, inspired by Swart's proposed proof and its revisions, Mihalis Yannakakis has recently shown that it is impossible to express the N-city traveling salesman polytope as the projection of a symmetric linear program of size polynomially bounded in N [43]. Also, those with ideas for proofs will be able to check with the repository to see if their proposed approach has been tried before.

Note, however, that this Repository is just that, a repository. We are not offering a verification service to new claimants. (It would of course be a conflict of interest for Mike Garey or myself to be involved in authenticating claims, given that our interest in the continuing sales of [G&J] provides a strong incentive for preferring one answer to the other.) We will, however, stamp each new proof with the date received, and thus may be of use in helping you establish priority, should your proof be correct. We will also maintain a list of the currently unperforated claims, and will be prepared, given their authors' permission, to send out copies of the proofs to any would-be verifiers that volunteer. (For a suggestion as to how the pool of would-be volunteers might be increased, given the perceived unlikelihood that any proposed proof will be correct, see [23].)

2. UPDATES, WE GET UPDATES

For those interested, the Repository currently occupies just two inches of file space in a five-drawer filing cabinet, the rest of which is devoted to as-yet-uncited material for this column. Much of this can be classified according to projected future column topics, such as ''Polyhedral Complexity,'' ''The complexity of Real Numbers,'' ''Interactive Complexity Classes,'' ''Database Complexity,'' etc. A significant portion, however, relates back to past columns by extending cited results, answering stated open problems, and correcting past imprecisions and errors. Of this, the material relating to Columns 1 through 14 will be included when those columns are republished in book form. Updates to Column 18, ''Computing in the Math Department, Part I,'' will be included when I get around to Part II, and Column 19 has not yet appeared as I write this. The remaining three columns, however, have more than enough updates to go around.

Column 15 was devoted to the ''complexity of uniqueness.'' A simpler proof has since been found for the central result of Valiant and Vazirani that SATISFIABILITY, restricted to those instances that have no more than one satisfying truth assignment, remains complete for NP under randomized reductions. The new proof is contained as a corollary to the main result in a paper on parallel complexity by Mulmuley, Vazirani, and Vazirani [29].

Another topic raised in Column 15 was the class D^p of languages that could be expressed as the intersection of a language L in NP and a language L' in co-NP. One type of problem contained in D^p is the ''criticality'' problem, for instance, MINIMAL UNSATISFIABILITY: Given a conjunctive normal form expression E (an instance of SATISFIABILITY), is it true that E is unsatisfiable, but that deleting any single clause from E yields a satisfiable expression? Interest in such criticality problems (in particular the ''TSP FACETS'' problem of [31]), and the thought that they might be complete for this class, were major inspirations for the original definition of D^p. It was thus somewhat frustrating to the experts that, at the time of Column 15, no criticality problems were known to be D^p-complete (although other types of problems *were* known to be D^p-complete, for which see the column). Fortunately, this source of frustration has now been removed by Papadimitriou and Wolfe, who in [30] have shown, via a very intricate construction, that MINIMAL UNSATISFIABILITY is D^p-complete. Using this, they prove that the same holds for MAXIMUM NON-HAMILTONIAN GRAPH, and hence, by an already-known reduction from [31], for TSP FACETS. (The definition of the latter, along with a discussion of its significance, will be postponed to the future column on polyhedral complexity.) Subsequent results of this form have also been proved for ''CRITICAL CLIQUE'' (by V. Vazirani, reported in [30]) and for ''CRITICAL UNCOLORABILITY'' (by Cai and Meyer in [12]).

Note that the class D^p is included in the Δ_2^p, the class of all languages recognizable in polynomial time given an oracle for SATISFIABILITY. Here, however, only two calls on the oracle are needed. Inspired by a private communication from Mark Krentel, Column 15 mused on the analogue of Δ_2^p for function computations (rather than languages), raising the question of whether this class could be separated into a hierarchy, based on how many calls to the oracle were allowed. For instance, consider the problem of computing the length of the shortest tour in an instance of the traveling salesman problem, for which a binary search algorithm would require a polynomial number of calls to the oracle. Could this be harder than computing the size of a maximum clique, for which binary search requires only $O(\log n)$ queries?

Krentel has now shown that, assuming P ≠ NP, the answer is yes [26]. The former function is complete for the class $\mathrm{FP}^{SAT}[n^{O(1)}]$ of all functions computable using a polynomial number of queries, the latter function is complete for the class $\mathrm{FP}^{SAT}[O(\log n)]$ of all functions computable using $O(\log n)$ queries, and $\mathrm{FP}^{SAT}[O(\log n)]$ is a proper subclass of $\mathrm{FP}^{SAT}[n^{O(1)}]$, assuming P ≠ NP. Furthermore, via a cute observation, Krentel shows that the bin packing problem is still easier: As remarked in Column 4, Karmarkar and Karp [24] have devised a polynomial-time algorithm for bin packing that is guaranteed to use no more than $OPT(L) + O(\log^2 OPT(L))$ bins. Thus, by first invoking the algorithm, one can cut the range of possible values down to $O(\log^2 n)$, and binary search will require only $O(\log \log n)$ queries. Since $FP^{SAT}[O(\log \log n)]$ is strictly contained in $FP^{SAT}[O(\log n)]$, again assuming P ≠ NP, the result follows [26]. The hope that such separation results might carry over easily from functions to languages is not bright, however, as the relativization evidence is against it. The well-known oracle set A that yields $\mathrm{P}^A \neq \mathrm{NP}^A = \mathrm{co-NP}^A$ [4] collapses all of $\Delta_2^{p,A}$ down to NP^A, and hence to the set of languages recognized by just one oracle call [26].

Column 16, devoted to the complexity of graph-theoretic problems for various classes of graphs, made the serious mistake of summarizing the current state of knowledge by a two-dimensional array (columns labeled by 10 basic problems and rows by 30 different graph classes). As anyone who has ever performed a similar exercise knows, the resulting array is almost certain to contain lots of question marks denoting problems of unknown complexity. I thus at one fell swoop created 70 ‘‘Open Problems of the Month,’’ and the next few paragraphs will no doubt be only the first of many attempts to clear up the debris.

Of those 70 problems, 17 were marked with the symbol ‘‘P?’’ to indicate that they appeared to be solvable in polynomial time by standard techniques, although I hadn’t checked the details. I still haven’t checked the details, but others have, and I was right. Indeed, for several of these no new algorithms were needed, as I had failed to make a basic graph-theoretic observation about partial k-trees. Although I did note that all series-parallel graphs (and hence all outerplanar graphs) are partial 2-trees, I failed to observe, as has now been shown in [7], that all Halin graphs are partial 5-trees, and that for any fixed k, all k-outerplanar graphs, all bandwidth-k graphs, and all ‘‘almost tree (k)’’ graphs are partial k'-trees for some fixed, possible larger values k'. Thus any problem solvable in polynomial time for partial k-trees when k is fixed is also in P for the other classes (when k, if relevant, is also fixed).

Thus, we only needed polynomial-time algorithms for partial k-trees to resolve all the open problems for the other classes. In five cases, we already had them (and this suffices to turn three of the P?’s into P’s). Thanks to Hans Bodlaender [8,9], we now have them for the other five, thus turning the remaining P?’s to P’s (as well as three O?’s, where the latter shorthand stood for ‘‘apparently open, but not certifiably hard’’). Polynomial-time algorithms for PARTITION INTO CLIQUES, MAX CUT (even in the weighted case), and (unweighted) STEINER TREE IN GRAPHS are presented in [8], as applications of a powerful new sufficient condition for the membership in P of problems restricted to partial k-trees, k fixed. (If, for fixed k and k', we simultaneously restrict our graphs to be partial k-trees and to obey a degree bound k', then weaker conditions suffice to imply membership in P [8,34,35] and indeed linear-time solvability [34,35].) The remaining two problems, CHROMATIC INDEX and GRAPH ISOMORPHISM, both marked O?, fall to the more powerful techniques in [9]. (The algorithm for CHROMATIC INDEX, though polynomial for fixed k, is doubly exponential in k, as opposed to a single exponential for all the other algorithms mentioned.)

The removal of five additional, isolated O?’s serves to round out the current picture. First, Bodlaender has shown that MAX CUT (the unweighted version, as specified in the table) is in P for cographs, although it becomes NP-complete for such graphs if even just weights 1 and 2 are allowed [10]. Second, Kouhei

Asano has provided me with a simple construction showing that GRAPH ISOMORPHISM restricted to thickness-k graphs for any fixed $k \geq 2$ is equivalent to general GRAPH ISOMORPHISM and hence deserves an ''I'' entry rather than an ''O?'' [3]. Third, Paul Dietz, in a Ph.D. thesis [16] that preceded the column but only recently came to my attention, has shown that isomorphism of directed path graphs is in P. Fourth, Bertossi and Bonuccelli [6] have shown that HAMILTONIAN CIRCUIT remains NP-complete for undirected path graphs. Finally (but actually firstly, since he made this observation to me one day after I mailed the final copy for Column 16 to Academic Press), Mihalis Yannakakis has observed that since the complement of any triangle-free graph is claw-free, and since INDEPENDENT SET remains NP-complete for triangle-free graphs (see [G&J]), CLIQUE is NP-complete for claw-free graphs [42]. (It was solvable in polynomial time for all 29 other graph restrictions.)

Another question addressed in Column 16 was whether, for the given collection of graph types, strict containment could always be illustrated by a problem that was hard for the larger class and easy for the smaller. One of my claimed illustrations turns out to have been in error: the proof in [39] that INDUCED SUBGRAPH ISOMORPHISM is NP-complete for series-parallel graphs makes the common error of attempting to use PARTITION in a pseudo-polynomial transformation (as does the proof of Proposition 3 in [38], another paper cited in Column 16). To make up for the loss of this separating result, N. Chandresekharan has pointed out to me that one of the pairs I missed *can* be separated: As shown in [15], the following ''k-CLUSTER'' problem is NP-complete for chordal graphs but solvable in polynomial time for split graphs: ''Given a graph G and an integer B, is there an induced subgraph of G with k vertices and B or more edges?''

Column 17 was devoted to lower bounds for circuit complexity, and I have one update for each of its sections. The first section was devoted to Razborov's superpolynomial lower bounds on the monotone circuit complexity of CLIQUE and PERFECT MATCHING, the first NP-complete and the second in P. I observed that although the lower bound for the former had been pushed to a full exponential by Alon and Boppana [1], the lower bound for the latter remained technically subexponential, leaving open the possibility that monotone circuit complexity might still separate P from NP, at least to a limited degree. That hope has now been dashed by Eva Tardos. In [40], she invents a monotone problem related to computing the ''Shannon capacity'' of a graph (as defined by Lovász in [27]), observes that the Alon-Boppana techniques apply to this problem to yield a true exponential lower bound, and then shows that the problem can be solved in polynomial time using ellipsoid method techniques from [22].

Section 2 concerned lower bounds for constant-depth unbounded fan-in circuits, and described Andy Yao's major result that circuits of this type that compute the parity function require an exponential number of AND and OR gates. Note that if one in addition had a ''parity'' gate, i.e., one that told whether an even number of its inputs were TRUE, polynomial-size, constant depth circuits for parity would be trivial to construct. A natural question was thus whether such gates might be of use in making small circuits for *other* problems that require exponentially many gates when restricted to AND's, OR's and NOT's, such as the majority function. Recently, Razborov has proved this *not* to be the case. Even with parity gates, majority requires an exponential number of AND's and OR's when computed with constant depth, unbounded fan-in circuits [33]. Even more interestingly, suppose we generalize the concept of the parity gate to that of a ''$\bmod\ p$'' gate, p prime, where such a gate outputs TRUE if and only if the number of its inputs currently set to TRUE is congruent to $0 \pmod p$. Extending the new algebraic techniques of [33], Roman Smolensky has shown that for any prime p and any positive integer r that is not a power of p, the computation of the $\bmod\ r$ function by constant depth, unbounded fan-in circuits requires an exponential number of gates even if $\bmod\ p$ gates are allowed [36]. (Moreover, such gates are no more helpful in computing majority than was the parity gate.) In addition, Smolensky's techniques simplify the proofs of the old results for parity and majority when only AND's, OR's and NOT's are allowed.

The final section of Column 17 emphasized constant *width*, rather than depth. I reported that Barrington's result about bounded-width ''branching programs'' could be interpreted as saying that polynomial-size circuits of width 5 could compute everything in NC^1 (the set of problems solvable by bounded fan-in circuits of polynomial size and logarithmic depth). In [5], Barrington shows that width 4 suffices.

In addition to the above remarks, I also note that the following references have now appeared in final journal form: [19,41] from Column 15, [2,17,20,21,25,32] from Column 16, and [11] from Column 17.

3. UNPUBLISHED NOTES FROM ALL OVER

Unfortunately, not all references cited in past editions of this column (and indeed in [G&J]) are likely to make it into the abovementioned ''final journal form.'' Thus the second of my two filing cabinets devoted to the column (the one containing already-cited references) has become near irreplaceable, a fact that is brought home to me each time I get a request for the details of the proof of some unpublished result that I have cited and long since forgotten.

Of those ''unpublished manuscripts'' and ''personal communications'' that have not yet seen the formal light of day, two in particular stand out. They were both originally cited in [G&J], and between them they seem to have garnered more enquiries than all the others combined, sending me off to the copier repeatedly to fulfill requests. One was the 1978 manuscript ''Some NP-complete set covering problems,'' by William Masek, who was then at MIT but has since disappeared from the theory community.

That paper contains the NP-completeness proofs for two problems of major importance in circuit and VLSI design. One was MINIMUM DISJUNCTIVE NORMAL FORM ([LO9] in [G&J]), where one is given the complete truth table for a Boolean function and asked if there is a disjunctive normal form expression for it whose size is less than a given bound. The second, related NP-completeness result was for the problem RECTILINEAR PICTURE COMPRESSION ([SR25] in [G&J]): Given a rectilinear polygon R, possibly containing holes, and an integer K, can R be expressed as the (non-disjoint) union of K or fewer rectangles? This problem has applications to VLSI mask production, as well as the display of figures on video terminals, and these applications perhaps account for the paper's popularity. (Since [G&J] appeared, polynomial-time algorithms have been discovered for the case where R is ''rectilinearly convex'' [13] and slight generalizations thereof [18], and NP-completeness has been proved for the restriction of the original problem to instances in ''general position'' [14], but the problem remains open if the only restriction is that R contain no holes. For results on related problems, see [14] and Column 3 [June 1982].)

Before disappearing, Masek sent me a revised manuscript [28], but as far as I know the paper was never published. If you are reading these words, Bill, wherever you are, just send word that you would like the paper to be considered as a submission to *J. Algorithms*, and I'll take it from there. Meanwhile, I can take more direct action on the second of our two ''greatest unpublished hits,'' as the authors here were Mike Garey and myself, and I have a page or so left in the column.

Our result concerned a problem that would not seem to have the practical significance of Masek's problems, but apparently has a certain basic appeal: the BALANCED COMPLETE BIPARTITE SUBGRAPH problem ([GT24] in [G&J]), in which one is given a connected bipartite graph G and an integer k and is asked whether G contains a complete bipartite subgraph with k vertices on each side. Although a hint is provided in [G&J] (''transformation from CLIQUE''), the NP-completeness proof seems to have been sufficiently subtle to have eluded many who tried to regenerate it. It provides another example of what might be called the ''critical balance'' proof technique, and is presented below. (As a warm up, readers might first try to see why the problem is solvable in polynomial time if we don't require the complete bipartite subgraph to be balanced, but only ask that it contain at least k vertices. Hint: show that the revised problem can be reduced to that of finding a maximum independent set in a bipartite graph, and show how the latter can be solved using matching techniques.)

Here is our transformation. Let $G = (V,E)$ and K provide an instance of CLIQUE. We may assume without loss of generality that $K = |V|/2$ (see the comments for [GT19] in [G&J]). We construct an instance $G' = (V',E')$, K' of

BALANCED COMPLETE BIPARTITE SUBGRAPH as follows: Let

$$V' = V \cup E \cup W,$$

where W is a set of $K(K-1)/2 - K$ new elements.

$$E' = \{\{e,w\}: e \in E,\, w \in W\}$$

$$\cup\ \{\{e,v\}: e \in E,\, v \text{ is } \textit{not} \text{ an endpoint of } e\}$$

$$K' = \binom{K}{2} = K(K-1)/2$$

This construction can clearly be performed in polynomial time. Note that the two ‘‘sides’’ of the bipartite graph are E and $V \cup W$, and that any complete bipartite subgraph can contain all of W. By our choice of K', we are asking whether it can in addition contain a set V_B of K elements from V and a set E_B of $K(K-1)/2$ members of E. By the definition of E' and the fact that our subgraph must be complete, no edge in E_B can have an endpoint in V_B, so all endpoints must lie in $V - V_B$. But then $(V - V_B, E_B)$ is a subgraph of G with K vertices and $K(K-1)/2$ edges, and the only possible such subgraph is a clique. So G' has a balanced complete bipartite subgraph of the desired size only if G has the desired clique. Similarly, if the desired clique (V_C, E_C) exists, then the desired bipartite subgraph will be $(W \cup V - V_C, E_C)$. Thus our construction is indeed a polynomial transformation, the proof is complete, and so is this column. □

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