

The NP-Completeness Column: An Ongoing Guide

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This is the 22nd edition of an (allegedly) quarterly column that covers new developments in the theory of NP-completeness. The presentation is modeled on that used by M. R. Garey and myself in our book "Computers and Intractability: A Guide to the Theory of NP-Completeness," W. H. Freeman & Co., New York, 1979 (hereinafter referred to as "[G&J]"; previous columns will be referred to by their dates). A background equivalent to that provided by [G&J] is assumed, and, when appropriate, cross-references will be given to that book and the list of problems (NP-complete and harder) presented there. Readers who have results they would like mentioned (NP-hardness, PSPACE-hardness, polynomial-time-solvability, etc.) or open problems they would like publicized, should send them to David S. Johnson, Room 2D-150, AT&T Bell Laboratories, Murray Hill, NJ 07974 (or to dsj@research.att.com). Please include details, or at least sketches, of any new proofs; full papers are preferred. If the results are unpublished, please state explicitly that you are willing for them to be mentioned. Comments and corrections are also welcome. For more details, see the December 1981 issue of this Journal.

THE STORY SO FAR

This will be a short column, written in response to the many requests I have received for a directory to the contents and locations of past editions. The column has been in hiatus for the last year and a half due to other projects, but I hope to be able to resume the publication of full-length columns soon, perhaps with the September, 1990 issue. There certainly has been no shortage of exciting new results to report while I've been away, and I am eager to get to them.

I am also eager to get to my long-promised "Collected Columns" book, but that seems to be even further away on the horizon. For the time being, therefore, readers interested in the material covered in earlier columns will have to access the columns directly. The following summaries should help in determining where various topics were discussed, and thus may reduce the amount of time spent thumbing through back issues of the journal.

I will cover each column in sequence, listing the issue and page numbers for it, followed by a descriptive title and a summary of the major topics covered, including the names of all displayed NP-hard problems and "open problems of

the month.’’ (Note that not all the columns had titles when they appeared, so in many cases the title is an after-the-fact invention.) Where appropriate, I will also include a few explanatory remarks and cross-references. Readers should also be aware that many of the columns also contain minor updates to related problems in [G&J] and in previous columns.

Column 1: “The Twelve Open Problems from [G&J]: Updates”

J. Algorithms 2:4 (1981), 393-405.

At the time of this column, six of the twelve had been resolved:

CHORDAL GRAPH COMPLETION (NP-complete)
CHROMATIC INDEX (NP-complete)
SPANNING TREE PARITY (in P)
PARTIAL ORDER DIMENSION (NP-complete)
LINEAR PROGRAMMING (in P)
TOTAL UNIMODULARITY (in P)

Subsequently, two more have been resolved (see Columns 19, 21):

SUBGRAPH HOMEOMORPHISM (FOR FIXED GRAPH H) (in P)
GRAPH GENUS (NP-complete)

Still remaining open:

GRAPH ISOMORPHISM
PRECEDENCE CONSTRAINED 3-PROCESSOR SCHEDULING
COMPOSITE NUMBER (but see Column 18)
MINIMUM-LENGTH TRIANGULATION

Open problem of the month (and still open): IMPERFECT GRAPH.

Column 2: “Embedding Problems”

J. Algorithms 3:1 (1982), 89-99.

A list of NP-hard questions about embedding graphs in other objects:

GRAPH ENCODABILITY
UNIFORM GRAPH ENCODABILITY
SUBGRAPH HOMOMORPHISM
EMBEDDING DIMENSION
WEIGHTED GRAPH EMBEDDABILITY
MINIMUM AREA EMBEDDING OF PLANAR GRAPHS
WEIGHTED TREE LAYOUT WITH FIXED LEAVES
EUMORPHOUS TREE LAYOUT
CROSSING NUMBER

The open problem of the month was GRAPH THICKNESS, which subsequently turned out to be NP-complete (see Column 5).

Column 3: “Partitioning, Covering, and Packing Problems”

J. Algorithms **3:2** (1982), 182-195.

A list of NP-hard questions about partitioning, covering, and packing:

EDGE PARTITION INTO TRIANGLES
DISTANCE-d CHROMATIC NUMBER
DISTANCE-d PARTITION OF POINTS IN THE PLANE
GEOMETRIC COVERING BY DISCS
INTEGRITY PRESERVING CUT
CUT INTO CONNECTED COMPONENTS OF BOUNDED SIZE
CUT INTO ACYCLIC SUBGRAPHS
 K^{th} BEST s-t CUT
WEAK PARTITION
GEOMETRIC PARTITION INTO TRIANGLE
PARTITION INTO CONVEX REGION
MINIMUM PERIMETER PARTITION INTO RECTANGLES
COVERING BY CONVEX POLYGONS
PACKING WITH SQUARES

Open problem of the month (and still open): EVEN COVER.

Column 4: “Oracles, Bin Packing, and Ordering Problems”

J. Algorithms **3:3** (1982), 288-300.

- (1) “Random” oracles and the downfall of the Random Oracle Hypothesis.
- (2) An asymptotic approximation scheme for one-dimensional bin packing.
- (3) New NP-hard problems about orderings and permutations:

GRAPH ISOMORPHISM WITH RESTRICTIONS
PARTITIONED GRAPH ISOMORPHISM
CYCLIC BANDWIDTH
SEARCH NUMBER
MINIMUM LENGTH GENERATOR SEQUENCE
SHUFFLED STRING
DEFECTIVE SORTING NETWORK

Column 5: “Routing Problems”

J. Algorithms **3:4** (1982), 381-395.

NP-hard problems with some connection, possibly tenuous, to routing:

BUS ROUTING
ROUTING WITH “MUST PAIRS”
EVEN LENGTH PATH
DELIVERY VAN ROUTING
EUCLIDEAN POINT COVER
PLANAR INTERCONNECTION ON A GRID
TWO-LAYER MULTIPLE MODULE ROUTING

TWO-LAYER CHANNEL ROUTING
SINGLE ROW ROUTING
GRAPH THICKNESS

A claimed result for MINIMUM-LENGTH PSEUDO-TRIANGULATION was subsequently withdrawn (see Column 10). Open problem of the month (and still open) “UNRESTRICTED” TWO-LAYER CHANNEL ROUTING.

Column 6: “Knapsacks, Linear Programs, and a Gamut of Problems”

J. Algorithms 4:1 (1983), 87-100.

- (1) Shamir’s cracking of the Merkle-Hellman knapsack cryptosystem.
- (2) Average case analysis of simplex algorithms for linear programming.
- (3) A foursome of unrelated NP-hardness results:

SIMULTANEOUS DIOPHANTINE APPROXIMATION
SINGLE-LETTER CONTEXT-FREE LANGUAGE MEMBERSHIP
GROUND STATE OF A SPIN GLASS
ROBOT ARM REACHABILITY

Open problem of the month (still open): GRACEFUL GRAPH.

Column 7: “Parallel Processing”

J. Algorithms 4:2 (1983), 189-203.

- (1) Trading hardware for time: PRAM’s, NC, SC, and POLYLOGSPACE.
- (2) NP-hard problems related to scheduling parallel processors:

MULTIPLE CHOICE SCHEDULING WITHIN INTERVALS
SCHEDULING TO MINIMIZE MACHINE COST
PREEMPTIVE PERIODIC SCHEDULING
MINIMUM COMPLETE RESEARCH PROBLEM

Column 8: “Concurrent Processing”

J. Algorithms 4:3 (1983), 286-300.

NP-hard questions about cooperating parallel processes:

DEADLOCK AVOIDANCE
DEADLOCK POTENTIAL
DEADLOCK RECOVERY
STRICT SERIALIZABILITY
MULTIVERSION SERIALIZABILITY
UNSAFE LOCKING POLICY
MINIMUM DEPTH D-SERIALIZABLE PARTIAL ORDER
DISTRIBUTED SERIALIZABILITY ASSURANCE

Column 9: “Fun and Games”

J. Algorithms 4:4 (1983), 397-411.

NP-hard questions about puzzles, games, and alternation:

SLIDING BLOCK REARRANGEABILITY
PARTIAL LATIN SQUARE EXTENDABILITY
THE PEBBLE GAME
YET ANOTHER PEBBLE GAME
GENERALIZED DOTS AND BOXES
PEEK
BLIND PURSUIT
DYNAMIC NETWORK RELIABILITY

Column 10: “Updates, Updates, Updates”

J. Algorithms **5:1** (1984), 147-160.

- (1) Updates to results from previous columns.
- (2) 20+ updates to the DOMINATING SET problem from [G&J].

Column 11: “Solving NP-Hard Problems Quickly (On Average)”

J. Algorithms **5:2** (1984), 284-299.

- (1) A variety of algorithms that “almost always” run in polynomial time.
- (2) Levin’s theory of average case complexity and a complete problem for “random NP” (RANDOM TILING).

Column 12: “Randomized Algorithms and Complexity Classes”

J. Algorithms **5:3** (1984), 433-447.

- (1) Situations where randomized algorithms do better than deterministic ones.
- (2) Probabilistic Turing Machines and the classes they define (ZPP, R, BPP, and PP).
- (3) Randomized reducibilities and a problem “R-complete” for NP (ENCODING BY TM - see Column 13 for a Corrigendum).

Column 13: “Communicating Processes”

J. Algorithms **5:4** (1984), 595-609.

- (1) Simulating computation with communication.
- (2) Communication as a complexity measure (and the NP-complete COMMUNICATION COMPLEXITY problem).
- (3) NP-hard problems about communication networks:

FILE TRANSFER SCHEDULING
DEADLOCK EXPOSURE IN PACKET SWITCHING NETWORKS
COMMUNICATION LINK TESTING
COMMUNICATION NODE TESTING

Column 14: “Network Design”

J. Algorithms **6:1** (1985), 145-159.

- NP-hard problems involved in designing and analyzing networks:

MINIMUM MAX-FLOW SPANNING TREE

MINIMUM CYCLE BASIS SPANNING TREE
BOUNDED DIAMETER DECOMPOSITION
DEGREE CONSTRAINED SUBGRAPH
MAXIMUM WEIGHT CONNECTED SUBGRAPH
NETWORK RELIABILITY UNDER UNCERTAINTY
RACE DETECTION
DEFECTIVE SUPERCONCENTRATOR
MULTIWAY SEARCH TREE COST MINIMIZATION
PLANAR VENN DIAGRAM

Column 15: “Uniqueness”

J. Algorithms **6:2** (1985), 291-305.

- (1) Valiant and Vazirani’s result about the difficulty of finding unique solutions.
- (2) Verifying uniqueness and the class D^P .
- (3) Computing with uniqueness: “unambiguous” Turing machines and the class UP.

Column 16: “Graph Restrictions and Their Effect”

J. Algorithms **6:3** (1985), 434-451.

30 different restrictions one can impose on undirected graphs, and their effect on the complexity of 9 important NP-complete graph problems and GRAPH ISOMORPHISM. Solutions to the problems left open in the resulting 300-entry table are still pouring in (see Column 20).

Column 17: “Computing with One Hand Tied behind Your Back”

J. Algorithms **7:2** (1986), 289-305.

Results about the computational power of severely restricted machines:

- (1) Razborov’s exponential lower bounds for monotone circuits.
- (2) Yao’s exponential lower bounds for bounded depth circuits computing PARITY, and the consequences for the relativized polynomial time hierarchy.
- (3) Barrington’s result that any polynomial size branching program can be simulated by one with width 5.

Column 18: “Computing in the Math Department, Part I”

J. Algorithms **7:4** (1986), 584-601.

- (1) The Goldwasser-Kilian algorithm based on elliptic curves that can verify primality in random polynomial-time for almost all primes, plus the extension to *all* primes by Adleman and Huang, implying COMPOSITE NUMBER is in ZPP.
- (2) Other number theoretic algorithms, complexity results, and open problems.
- (3) Algorithms and complexity results related to factoring polynomials.
- (4) Open problem of the month (still open): SHORTEST VECTOR IN A LATTICE.

Column 19: “The Many Faces of Polynomial Time”

J. Algorithms **8:2** (1987), 285-303.

- (1) Robertson and Seymour’s polynomial time algorithms for K-PATH PROBLEM (FOR FIXED K) and SUBGRAPH HOMEOMORPHISM (FOR FIXED GRAPH H).
- (2) How the above algorithms, plus the Robertson-Seymour proof of Wagner’s Conjecture, can yield non-constructive proofs of membership in P.
- (3) Strongly polynomial time algorithms.

Column 20: “Announcements, Updates, and Greatest Hits”

J. Algorithms **8:3** (1987), 438-448.

- (1) False proofs that $P = NP$ (or $P \neq NP$) and what to do with them.
- (2) Updates on Columns 15, 16, and 17.
- (3) Sketch of one of the most-requested unpublished NP-completeness proofs cited in [G&J] (for BALANCED COMPLETE BIPARTITE SUBGRAPH).

Column 21: “Interactive Proof Systems for Fun and Profit”

J. Algorithms **9:3** (1988), 426-444.

- (1) Interactive proof systems and the classes they generate (MA, AM, and AM[poly]).
- (2) Zero-knowledge proofs.
- (3) The NP-completeness of the GENUS problem (see Column 1).

Column 22: “The Story So Far”

J. Algorithms **11:1** (1989), this issue.

- (1) Summary of the contents of the first 22 NP-completeness columns.
- (2) Other scholarly publications, past and future, that may be of interest.

As indicated by the final self-referential listing above, I shall conclude this column with a brief description of some other works that may be of interest to readers of this column. Two of these are projects of my own, and share the blame for the recent long gap between column appearances.

First, there is a paper entitled “A Catalogue of Complexity Classes,” which is to appear in the *Handbook of Theoretical Computer Science, Volume A*, due from North-Holland in June 1990. This survey is as long as six columns put together and runs the gamut from deterministic log time to the non-elementary functions. Much of this material has been, or will be covered in the column, but in the meantime, copies of the survey are available on request.

Second, there is the *STOC-FOCS Bibliography*, an annotated listing of all the papers that have appeared since 1961 in the proceedings of the ACM Symposium on Theory of Computing (STOC) and the IEEE Symposium on Foundations of Computer Science and its predecessors (FOCS). The annotations have been provided by the authors, and indicate corresponding journal papers, if any, as well as other updates and corrigenda. This project is nearing completion

(updates on some 70% of the almost 2,000 proceedings papers have been received). The usefulness of such a bibliography should be evident to regular readers of this column, given the large number of STOC and FOCS papers that typically show up in each column's reference list. The bibliography will be available from ACM by summer 1990, and will be mailed to all members of the ACM Special Interest Group on Automata and Computability Theory (SIGACT).

Finally, I would like to direct attention to another regularly-appearing column, and to a significant historical discovery recently reported therein. Juris Hartmanis has for the past few years been writing a "Structural Complexity Column" in the *Bulletin of the EATCS* (see [1-7]). I suspect that many readers will find these columns interesting, but I would like to draw particular attention to the June 1989 edition [7], which looks at the early history of the P versus NP question. Although the precise definitions of the classes involved came later, it turns out that at least one famous mathematician was speculating about the issues involved as far back as 1956. Kurt Gödel, in a recently discovered letter to John von Neumann, observed that checking a proof appeared to be much easier than finding one, and remarked at the wonderful consequences that would follow if, say, one could find proofs of length n in $O(n^2)$ time (i.e., if $P = NP$). See the column [7] for more details and some entertaining speculations.

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7. J. HARTMANIS, The structural complexity column: Gödel, von Neumann and the $P = ?$ NP Problem, *Bull. European Assoc. Theoret. Comput. Sci.* **38** (June, 1989), 101-107.