

AN ANNOTATED BIBLIOGRAPHY OF GRASP

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ABSTRACT. This paper presents an annotated bibliography of greedy randomized adaptive search procedures (GRASP). The bibliography is current up to early 2004. The bibliography contains: background material; tutorials and surveys; enhancements to the basic method; hybrid methods; software; parallel GRASP; graph theory; quadratic and other assignment problems; location, layout, and cutting; covering, clustering, packing, and partitioning; routing; sequencing and scheduling; logic; manufacturing; transportation; telecommunications; electrical power systems; biology; VLSI design; drawing; and miscellaneous topics. The collection includes papers published in journals, books, M.Sc. and Ph.D. dissertations, and unpublished technical reports. The online version of this bibliography is updated periodically and can be accessed at <http://www.graspheuristic.org>.

KEYWORDS. GRASP, metaheuristics, local search, combinatorial optimization.

1. INTRODUCTION

Optimization problems that involve a finite number of alternatives often arise in practice. Given a finite solution set X and a real-valued function $f : X \rightarrow \mathbb{R}$, one seeks a solution $x^* \in X$ with $f(x^*) \leq f(x)$, $\forall x \in X$. Common examples include designing efficient telecommunication networks, constructing cost effective airline crew schedules, and producing efficient routes for waste management pickup.

Much work has been done over the last five decades to develop optimal seeking methods that do not explicitly require an examination of each alternative. This research has given rise to the field of *combinatorial optimization* (see Papadimitriou and Steiglitz (1982)), and an increasing capability to solve ever larger real-world problems. Nevertheless, most problems found in practice are either computationally intractable by their nature, or sufficiently large so as to preclude the use of exact algorithms. In such cases, heuristic methods are usually employed to find good, but not necessarily guaranteed optimal solutions. The effectiveness of these methods depends upon their ability to adapt to a particular realization, avoid entrapment at local optima, and exploit the basic structure of the problem. Building on these notions, various heuristic search techniques have been developed that have demonstrably improved our ability to obtain good solutions to difficult combinatorial optimization problems. The most promising of such techniques include simulated annealing (Kirkpatrick, 1984), tabu search (Glover, 1989, 1990, Glover and Laguna, 1997), genetic algorithms (Goldberg, 1989), variable neighborhood search (Hansen and Mladenović, 1998), and GRASP (Greedy Randomized Adaptive Search Procedures) (Feo and Resende, 1989, 1995).

A GRASP is a multi-start or iterative process (Lin and Kernighan, 1973), in which each GRASP iteration consists of two phases, a construction phase, in which a feasible solution is produced, and a local search phase, in which a local optimum in the neighborhood of the constructed solution is sought. The best overall solution is kept as the result.

In the construction phase, a feasible solution is iteratively constructed, one element at a time. The basic GRASP construction phase is similar to the semi-greedy heuristic proposed independently by Hart and Shogan (1987). At each construction iteration, the choice of the next element to be added is determined by ordering all candidate elements (i.e. those that can be added to the solution) in a candidate list C with respect

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to a greedy function $g : C \rightarrow \mathbb{R}$. The heuristic is adaptive because the benefits associated with every element are updated at each iteration of the construction phase to reflect the changes brought on by the selection of the previous element. The probabilistic component of a GRASP is characterized by randomly choosing one of the best candidates in the list, but not necessarily the top candidate. The list of best candidates is called the *restricted candidate list* (RCL).

While local optimization procedures can require exponential time (Johnson et al., 1988) from an arbitrary starting point, empirically their efficiency significantly improves as the initial solution improves. The result is that often many GRASP solutions are generated in the same amount of time required for the local optimization procedure to converge from a single random start. Furthermore, the best of these GRASP solutions is generally significantly better than the single solution obtained from a random starting point.

An especially appealing characteristic of GRASP is the ease with which it can be implemented. Few parameters need to be set and tuned, and therefore development can focus on implementing efficient data structures to assure quick GRASP iterations. Finally, GRASP can be trivially implemented in parallel. Each processor can be initialized with its own copy of the procedure, the instance data, and an independent random number sequence. The GRASP iterations are then performed in parallel with only a single global variable required to store the best solution found over all processors.

In this article, we provide an annotated bibliography of the GRASP literature up to early 2004. This document contains references related to GRASP that have either appeared in the literature or as technical reports. We first look at tutorials and surveys. Papers that propose enhancements to the basic heuristic are considered next. Following that, we examine GRASP as a component of a hybrid metaheuristic. GRASP source code and parallelization of GRASP follow. The paper concludes with a literature review of operations research and computer science applications of GRASP as well as industrial applications. These include graph theory, quadratic and other assignment problems, location, layout, cutting, covering, clustering, packing, partitioning, routing, sequencing and scheduling, logic, manufacturing, transportation, telecommunications, electrical power systems, biology, VLSI design, drawing, and miscellaneous topics.

This bibliography is updated periodically and can be also accessed at <http://www.graspheuristic.org>. If the reader is aware of any uncited reference, incorrectly cited reference, or update to a cited reference, please contact the authors.

Feo, T. and M. Resende (1989). A probabilistic heuristic for a computationally difficult set covering problem. *Operations Research Letters* **8**, 67–71.

This is the first paper to explicitly describe a GRASP. See also page 27.

Feo, T. and M. Resende (1995). Greedy randomized adaptive search procedures. *J. of Global Optimization* **6**, 109–133.

An early survey of GRASP. See also page 3.

Glover, F. (1989). Tabu search – Part I. *ORSA J. on Computing* **1**, 190–206.

Description of the tabu search metaheuristic.

Glover, F. (1990). Tabu search – Part II. *ORSA J. on Computing* **2**, 4–32.

Description of the tabu search metaheuristic.

Glover, F. and M. Laguna (1997). *Tabu Search*. Kluwer Academic Publishers.

Book on metaheuristics and, in particular, tabu search.

Goldberg, D. (1989). *Genetic algorithms in search, optimization and machine learning*. Addison-Wesley.

Book on genetic algorithms.

Hansen, P. and N. Mladenović (1998). An introduction to variable neighborhood search. In S. Voss, S. Martello, I. H. Osman, and C. Roucairol (Eds.), *Meta-heuristics, Advances and trends in local search paradigms for optimization*, pp. 433–458. Kluwer Academic Publishers.

Description of the variable neighborhood search metaheuristic.

- Hart, J. and A. Shogan (1987). Semi-greedy heuristics: An empirical study. *Operations Research Letters* **6**, 107–114.
Presents a randomized greedy heuristic, called semi-greedy heuristic.
- Johnson, D., C. Papadimitriou, and M. Yannakakis (1988). How easy is local search? *Journal of Computer and System Sciences* **17**, 79–100.
Study of the computational complexity of local search.
- Kirkpatrick, S. (1984). Optimization by simulated annealing: Quantitative studies. *J. of Statistical Physics* **34**, 975–986.
Description of the simulated annealing metaheuristic.
- Lin, S. and B. Kernighan (1973). An effective heuristic algorithm for the traveling-salesman problem. *Operations Research* **21**, 498–516.
An early random multistart local search technique.
- Papadimitriou, C. and K. Steiglitz (1982). *Combinatorial optimization: Algorithms and complexity*. Prentice-Hall.
A classic book on combinatorial optimization.

2. TUTORIALS AND SURVEYS

Introductory materials, such as tutorials and surveys of GRASP, have appeared in several languages. These papers are listed below.

- Blum, C. and A. Roli (2003). Metaheuristics in combinatorial optimization: Overview and conceptual comparison. *ACM Computing Surveys* **35**(3), 268–308.
Survey of most important metaheuristics. Differences and similarities among them are highlighted and some advantages and disadvantages of each underlined. Concludes that it is important to design hybrids of metaheuristics.
- Feo, T. and M. Resende (1995). Greedy randomized adaptive search procedures. *Journal of Global Optimization* **6**, 109–133.
This tutorial paper defines the various components comprising a GRASP. A general trivial implementation of GRASP on a parallel computer is given. The GRASP literature until 1994 is surveyed. See also page 2.
- Festa, P. (2003). Greedy randomized adaptive search procedures. *AIROnews* **7**(4), 7–11.
The basic components of GRASP and its successful implementations are described. Improved and alternative solution construction mechanisms are discussed along with different techniques for speeding up the search, hybridizations with other metaheuristics, and intensification and post-optimization strategies using path-relinking.
- Festa, P. and M. Resende (2002). GRASP: An annotated bibliography. In C. Ribeiro and P. Hansen (Eds.), *Essays and surveys in metaheuristics*, pp. 325–367. Kluwer Academic Publishers.
An annotated bibliography of GRASP. Covers work related to GRASP that has either appeared in the literature or as technical reports on or before 2001.
- González, J. (1996). GRASP. In A. Díaz (Ed.), *Heuristic optimization and neural networks in operations management and engineering*, pp. 143–161. Madrid: Editorial Paraninfo.
Chapter on GRASP in a book on heuristic procedures for optimization. In Spanish.
- Jin-Kao, H., P. Galinier, and M. Habib (2000). Métaheuristiques pour l’optimisation combinatoire et l’affectation sous contraintes. *Revue d’Intelligence Artificielle* **13**(2), 283–324.
Highlights main properties of existing metaheuristics to help researchers to choose the most suitable one in practice. One of the metaheuristic addressed is GRASP. In French.
- Pitsoulis, L. and M. Resende (2002). Greedy randomized adaptive search procedures. In P. Pardalos and M. Resende (Eds.), *Handbook of Applied Optimization*, pp. 178–183. Oxford University Press.

Chapter with survey of GRASP. Multi-start heuristics are seen as a way to apply local search to solve combinatorial optimization problems. GRASP is shown to, in some ways, improve upon greedy or random multi-start procedures. Includes enhancements to GRASP, parallelization of GRASP, and a survey of GRASP for solving problems in logic, assignment, and location.

Resende, M. (2001). Greedy randomized adaptive search procedures (GRASP). In C. Floudas and P. Pardalos (Eds.), *Encyclopedia of Optimization*, Volume 2, pp. 373–382. Kluwer Academic Publishers.

A survey of GRASP. The basic GRASP is described in detail and enhancements to the basic procedure are given. Several applications of GRASP are reported.

Resende, M. and C. Ribeiro (2003a). GRASP and path-relinking: Recent advances and applications. In T. Ibaraki and Y. Yoshitomi (Eds.), *Proceedings of the Fifth Metaheuristics International Conference (MIC2003)*, pp. T6–1 – T6–6.

The basic components of GRASP and path-relinking, as well as their hybridization, are described. See also page 9.

Resende, M. and C. Ribeiro (2003b). GRASP and path-relinking: Recent advances and applications. Technical report, Internet and Network Systems Research Center, AT&T Labs Research, Florham Park, NJ.

This paper addresses recent advances and application of hybridizations of GRASP and path-relinking. See also page 9.

Resende, M. and C. Ribeiro (2003c). Greedy randomized adaptive search procedures. In F. Glover and G. Kochenberger (Eds.), *Handbook of Metaheuristics*, pp. 219–249. Kluwer Academic Publishers.

Survey of GRASP. The basic GRASP is explained in detail and enhancements to the basic procedure are described, including several hybridizations recently proposed in the literature. Several applications of GRASP are reported, showing how this method can find good approximate solutions to operations research problems and industrial applications.

Resende, M. and J. G. Velarde (2003). GRASP: Procedimientos de búsqueda miope aleatorizado y adaptativo (GRASP: Greedy randomized adaptive search procedures). *Inteligencia Artificial* 2, 61–76.

This paper surveys GRASP. It covers construction mechanisms, local search, the use of path-relinking within GRASP, parallel GRASP, and reviews recent applications of GRASP. In Spanish.

Ribeiro, C. (2002). GRASP: Une métaheuristique gloutonne et probabiliste. In J. Teghem and M. Pirlot (Eds.), *Optimisation approchée en recherche opérationnelle*, pp. 153–176. Hermès.

This chapter surveys GRASP. The basic method is described and its applications reviewed. Parallel strategies are discussed and the method’s hybridization is considered. In French.

Silveira, C. (1999). GRASP – Uma heurística para resolução de problemas de otimização combinatoria. Technical report, Institute of Informatics, Federal University of Rio Grande do Sul, Porto Alegre, RS, Brazil.

This report provides an exhaustive description of the features of GRASP as a metaheuristic method for solving hard combinatorial optimization problems. In Portuguese.

Yagiura, M. and T. Ibaraki (1996). Genetic and local search algorithms as robust and simple optimization tools. In *Meta-heuristics: Theory and applications*, pp. 63–82. Kluwer Academic Publishers.

Various metaheuristics such as random multi-start local search (MLS) and genetic algorithm (GA) are implemented and their performance compared. The objective is not to propose the most powerful technique, but to compare general tendencies of various algorithms. From their analysis, they conclude that a GRASP type modification of MLS improves its performance and that GA combined with local search is effective if long computational time is allowed.

3. ENHANCEMENTS TO BASIC METHOD

The basic GRASP framework repeatedly builds a greedy randomized solution and applies local improvement to it. Several enhancements to the basic GRASP method have been proposed. Such enhancements can be found in the following papers.

Binato, S. and G. Oliveira (2002). A Reactive GRASP for transmission network expansion planning. In C. Ribeiro and P. Hansen (Eds.), *Essays and surveys in metaheuristics*, pp. 81–100. Kluwer Academic Publishers.

The GRASP previously proposed by Binato, Oliveira, and Araújo (1998) for solving a transmission network expansion problem is enhanced with the reactive scheme of Prais and Ribeiro (2000). See also page 43.

Bresina, J. (1996). Heuristic-biased stochastic sampling. In *Proceedings of the AAAI-96*, pp. 271–278.

Presents a generalization of iterative sampling called Heuristic-Biased Stochastic Sampling (HBSS). HBSS uses a given search heuristic to focus its exploration. The degree of focusing is determined by a chosen *bias function* that reflects the confidence one has in the heuristic's accuracy. This methodology can be directly applied in a GRASP construction phase, by biasing the selection of RCL elements to favor those with higher greedy function values.

Canuto, S., M. Resende, and C. Ribeiro (2001). Local search with perturbation for the prize-collecting Steiner tree problems in graphs. *Networks* **38**, 50–58.

Several local search strategies are investigated for solving the prize-collecting Steiner tree problems in graphs, including path relinking, VNS, and a GRASP that uses cost perturbations. See also page 7, page 15, and page 39.

Fleurent, C. and F. Glover (1999). Improved constructive multistart strategies for the quadratic assignment problem using adaptive memory. *INFORMS Journal on Computing* **11**, 198–204.

This paper illustrates that constructive multistart methods, such as Random Restart and GRASP, can be improved by the addition of memory and associated heuristic search principles. See also page 19.

Laguna, M. and R. Martí (1999). GRASP and path relinking for 2-layer straight line crossing minimization. *INFORMS Journal on Computing* **11**, 44–52.

This paper proposes the GRASP with path-relinking hybridization to minimize straight line crossings in a 2-layer graph. See also page 8 and page 46.

Prais, M. (2000). *Estratégias de variação de parâmetros em procedimentos GRASP e aplicações*. Ph. D. thesis, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil.

A GRASP for a matrix decomposition problem in TDMA traffic assignment is described. Proposes Reactive GRASP, a refinement of GRASP where the RCL parameter is adjusted dynamically to favor values that produce good solutions. Reactive GRASP is compared with other RCL strategies on matrix decomposition, set covering, maximum satisfiability and graph planarization.

Prais, M. and C. Ribeiro (2000a). Reactive GRASP: An application to a matrix decomposition problem in TDMA traffic assignment. *INFORMS Journal on Computing* **12**, 164–176.

This paper describes a GRASP for matrix decomposition problem arising in the context of traffic assignment in communication satellites. A refinement of GRASP, called Reactive GRASP, is proposed. Instead of using a fixed value for the basic parameter that defines the restrictiveness of the candidate list during the construction phase, Reactive GRASP self-adjusts the parameter value according to the quality of the solutions previously found. The local search phase of the GRASP proposed is based on a new neighborhood, defined by constructive and destructive moves. See also page 21 and page 41.

Prais, M. and C. Ribeiro (2000b). Variação de parâmetros em procedimentos GRASP. *Investigación Operativa* **9**, 1–20.

Investigates four strategies for the variation of the GRASP RCL parameter α : 1) reactive – α is self-adjusted along the iterations; 2) uniform – α is randomly chosen from a discrete uniform probability distribution; 3) hybrid – α is randomly chosen from a fixed probability distribution concentrated around the value corresponding to the purely greedy choice; 4) fixed – α has a fixed value close to the purely greedy choice. The reactive strategy is the most flexible and adherent to the characteristics of the specific problem to be solved. In Portuguese.

Rangel, M., N. Abreu, and P. Boaventura Netto (2000). GRASP para o PQA: Um limite de aceitação para soluções iniciais. *Pesquisa Operacional* **20**, 45–58.

Presents a modified version of the GRASP for the quadratic assignment problem proposed by Li, Pardalos, and Resende (1994). The new GRASP uses a criterion to accept or reject a given initial solution, thus trying to avoid potentially fruitless searches. In Portuguese. See also page 21.

Ribeiro, C., E. Uchoa, and R. Werneck (2002). A hybrid GRASP with perturbations for the Steiner problem in graphs. *INFORMS Journal on Computing* **14**, 228–246.

A hybrid GRASP is proposed for the Steiner problem in graphs. The construction phase is replaced by either one of several different construction procedures that apply weight perturbation strategies combining intensification and diversification elements. An adaptive path-relinking procedure is used as a post-optimization strategy. See also page 9 and page 18.

4. HYBRID METHODS

GRASP has been hybridized with other metaheuristic frameworks, such as tabu search, simulated annealing, genetic algorithms, variable neighborhood search, and path-relinking. The papers in this section propose GRASP hybrids.

Abdinnour-Helm, S. and S. Hadley (2000). Tabu search based heuristics for multi-floor facility layout. *International Journal of Production Research* **38**, 365–383.

Two two-stage heuristics are proposed for solving the multi-floor facility layout problem. GRASP/TS applies a GRASP to find the initial layout and tabu search to refine the initial layout, based on total inter/intra-floor costs. See also page 23.

Ahmadi, S. and I. Osman (2003). Greedy random adaptive memory programming search for the capacitated clustering problem. *European Journal of Operational Research*. To appear.

In order to introduce memory into the pure GRASP framework, the authors propose a GRAMPS framework, which is an hybrid of GRASP and AMP (Adaptive Memory Programming). Local search procedure is descent based on a restricted λ -interchange neighborhood. See also page 26.

Ahuja, R., J. Orlin, and A. Tiwari (2000). A greedy genetic algorithm for the quadratic assignment problem. *Computers and Operations Research* **27**, 917–934.

A genetic algorithm for the QAP is proposed. It incorporates the construction phase of the GRASP for QAP of Li, Pardalos, and Resende (1994) to generate the initial population. See also page 18.

Aiex, R., S. Binato, and M. Resende (2003). Parallel GRASP with path-relinking for job shop scheduling. *Parallel Computing* **29**, 393–430.

A parallel GRASP with path-relinking for the job shop problem is described using some ideas proposed in the GRASP of Binato et al. (2002). See also page 11 and page 30.

Aiex, R. and M. Resende (2003). Parallel strategies for GRASP with path-relinking. Technical report, Internet and Network Systems Research Center, AT&T Labs Research, Florham Park, NJ.

This paper analyzes two parallel strategies for GRASP with path-relinking and proposes a criterion to predict parallel efficiency based on experiments with a sequential implementation of the algorithm. The 3-index assignment problem and the job-shop scheduling problem are studied. See also page 11.

Aiex, R., M. Resende, P. Pardalos, and G. Toraldo (2000). GRASP with path relinking for the three-index assignment problem. Technical report, AT&T Labs Research, Florham Park, NJ 07733. To appear in *INFORMS J. on Computing*.

This paper describes variants of GRASP with path relinking for the three index assignment problem (AP3). See also page 19.

Álvarez, A., J. L. González, and K. De-Alba (2003). Scatter search for a network design problem. Technical Report PISIS-2003-02, Universidad Autónoma de Nuevo León, Facultad de Ingeniería Mecánica y Eléctrica, División de Posgrado en Ingeniería de Sistemas, Mexico.

A scatter search (SS) algorithm for solving a fixed charge capacitated multicommodity network design problems on undirected networks is proposed. Different implementations of SS consist in using different subroutines, including a Diversification Generation Method (DGM). The DGM proposed in this paper is a GRASP with some memory features incorporated. See also page 38.

Alvarez-Valdés, R., A. Parajón, and J. Tamarit (2002). A tabu search algorithm for large-scale guillotine (un)constrained two-dimensional cutting problems. *Computers & Operations Research* **29**, 925–947.

This paper proposes several heuristics for the two-dimensional cutting problem. One algorithm is a tabu search procedure, while two further heuristics are simple constructive procedures based on bounds obtained by solving one-dimensional knapsack problems. Those two constructive procedures are embedded in a GRASP. The authors also implement a path-relinking procedure as a post-optimization phase. See also page 23.

Areibi, S. and A. Vannelli (2000). Efficient hybrid search techniques for circuit partitioning. In *IEEE 4th World Multiconference on Circuits, Systems, Communications & Computers*.

Simulated annealing, a tabu search, a GRASP, and a genetic algorithm are applied to circuit partitioning. Two further search techniques are also proposed as hybrids, where a GRASP and a genetic algorithm are used for generating good initial partitions. See also page 23, page 26, and page 45.

Cano, J., O. Cordón, F. Herrera, and L. Sánchez (2002). A GRASP algorithm for clustering. In F. J. Garijo, J. Santos, and M. Toro (Eds.), *Advances in Artificial Intelligence - IBERAMIA 2002, 8th Ibero-American Conference on AI, Seville, Spain, November 12-15, 2002, Proceedings*, Volume 2527 of *Lecture Notes in Computer Science*, pp. 214–223. Springer.

A GRASP for cluster analysis is described. Construction is done using a randomized greedy Kaufman procedure and local search uses the K -means algorithm. The best solutions are found with a hybrid GRASP/ K -means with Kaufman initialization. See also page 24 and page 27.

Canuto, S., M. Resende, and C. Ribeiro (2001). Local search with perturbation for the prize-collecting Steiner tree problems in graphs. *Networks* **38**, 50–58.

Local search strategies are investigated, including path relinking, VNS, and a GRASP with cost perturbations. The greedy choice takes into account the input edge weights, while the local search routine defines as neighborhood of a solution $T(X)$ the set of all minimum spanning trees $T(X')$, where X' differs from X for exactly one node. See also page 5, page 15, and page 39.

Carreto, C. and B. Baker (2002). A GRASP interactive approach to the vehicle routing problem with back-hauls. In C. Ribeiro and P. Hansen (Eds.), *Essays and surveys in metaheuristics*, pp. 185–200. Kluwer Academic Publishers.

The incorporation of interactive tools into heuristics is investigated. A GRASP is used in the route construction and improvement phase. Details of the GRASP implementation are given on page 29.

de Souza, M., C. Duhamel, and C. Ribeiro (2003). A GRASP heuristic for the capacitated minimum spanning tree problem using a memory-based local search strategy. In M. Resende and J. de Sousa (Eds.), *Metaheuristics: Computer decision-making*, pp. 627–658. Kluwer Academic Publishers.

A novel local search is proposed. It applies reduction techniques to speed up the search and uses a new path-based neighborhood structure, defined by path exchanges. Details are given on page 15.

Delmaire, H., J. Díaz, E. Fernández, and M. Ortega (1999). Reactive GRASP and tabu search based heuristics for the single source capacitated plant location problem. *INFOR* **37**, 194–225.

The single source capacitated plant location problem is a discrete location problem that takes into account capacities in the plants to be opened and imposes that clients be served from a single open plant. A hybrid GRASP is proposed. See also page 24.

Faria Jr., H., S. Binato, M. Resende, and D. Falcão (2004). Power transmission network design by greedy randomized adaptive path relinking. *IEEE Transactions on Power Systems*. To appear.

This paper applies GRASP concepts to path relinking, producing a new metaheuristic (greedy randomized adaptive path relinking) which is applied to solve static power transmission network design problems. See also page 43.

Festa, P., P. Pardalos, M. Resende, and C. Ribeiro (2002). Randomized heuristics for the MAX-CUT problem. *Optimization Methods and Software* **7**, 1033–1058.

The authors propose and test heuristics for the MAX-CUT problem, including a GRASP, a variable neighborhood search (VNS), path relinking, and new hybrid heuristics that combine GRASP, VNS, and path relinking. See also page 15.

Laguna, M. and R. Martí (1999). GRASP and path relinking for 2-layer straight line crossing minimization. *INFORMS Journal on Computing* **11**, 44–52.

This is the first paper to develop the hybrid GRASP with path relinking. See also page 5 and page 46.

Marinakis, Y. and A. Migdalas (2003). Expanding neighborhood GRASP for the Traveling Salesman Problem. Technical report, Technical University of Crete, Chania 73100, Greece.

A GRASP, that includes tour improvement methods, is proposed for the traveling salesman problem. See also page 16 and page 30.

Oliveira, C., P. Pardalos, and M. Resende (2003). GRASP with path-relinking for the QAP. In T. Ibaraki and Y. Yoshitomi (Eds.), *Proceedings of the Fifth Metaheuristics International Conference (MIC2003)*, pp. 57–1 – 57–6.

A GRASP using path relinking as intensification phase is proposed for the quadratic assignment problem. See also page 20.

Pacheco, J. and O. Valencia (2003). Design of hybrids for the minimum sum-of-squares clustering problem. *Computational Statistics and Data Analysis* **43**, 235–248.

For the non-hierarchical clustering problem under the criterion of minimum sum-of-squares clustering, a set of heuristics is proposed that incorporate genetic operators, local search, and tabu search. See also page 28.

Palubeckis, G. and A. Tomkevicius (2002). GRASP implementations for the unconstrained binary quadratic optimization problem. *Information Technology and Control* **24**, 14–20.

A classical GRASP framework and an enhanced GRASP that uses a simple tabu search as local search are proposed. See also page 49.

Piñana, E., I. Plana, V. Campos, and R. Martí (2004). GRASP and path relinking for the matrix bandwidth minimization. *European J. of Operational Research* **153**(1), 200–210.

Given matrix $A = \{a_{ij}\}_{n \times n}$, the matrix bandwidth minimization problem consists in finding a permutation of the rows and columns of A that keeps the nonzero elements in a band lying as close as possible to the main diagonal of A . The original problem is equivalent to the problem of finding a labeling f of the vertices that minimizes the maximum difference between labels of adjacent vertices. To solve the problem, a GRASP is proposed.

Resende, M. and C. Ribeiro (2003a). GRASP and path-relinking: Recent advances and applications. In T. Ibaraki and Y. Yoshitomi (Eds.), *Proceedings of the Fifth Metaheuristics International Conference (MIC2003)*, pp. T6–1 – T6–6.

Successful implementations of GRASP and path relinking are described in detail and several real world problems application are reported. See also page 4.

Resende, M. and C. Ribeiro (2003b). GRASP and path-relinking: Recent advances and applications. Technical report, Internet and Network Systems Research Center, AT&T Labs Research, Florham Park, NJ.

Recent advances and application of hybridizations of greedy randomized adaptive search procedures (GRASP) and path-relinking are addressed. A template for implementing path-relinking as an intensification procedure for GRASP is presented. Enhancements to the procedure, recently described in the literature, are reviewed. The effectiveness of the procedure is illustrated experimentally. See also page 4.

Resende, M. and C. Ribeiro (2003c). GRASP with path-relinking for private virtual circuit routing. *Networks* **41**, 104–114.

Variants of a GRASP with path-relinking heuristic are proposed for the offline PVC routing problem. See also page 30 and page 41.

Resende, M. and R. Werneck (2003). A hybrid multistart heuristic for the uncapacitated facility location problem. Technical report, Internet and Network Systems Research Center, AT&T Labs Research, Florham Park, NJ.

A multistart heuristic is described based on an idea proposed in Resende and Werneck (2004). See also page 25.

Resende, M. and R. Werneck (2004). A hybrid heuristic for the p -median problem. *Journal of Heuristics* **10**, 59–88.

A GRASP with path-relinking with a post-optimization phase is proposed for the p -median location problem. See also page 25.

Ribeiro, C., E. Uchoa, and R. Werneck (2002). A hybrid GRASP with perturbations for the Steiner problem in graphs. *INFORMS Journal on Computing* **14**, 228–246.

In this hybrid GRASP for the Steiner problem in graphs, the GRASP construction phase is replaced by either one of several different construction procedures that apply weight perturbation strategies combining intensification and diversification elements. An adaptive path-relinking procedure is used at the end as a post-optimization strategy. See also page 6 and page 18.

Ribeiro, C. and D. Vianna (2003). A GRASP/VND heuristic for the phylogeny problem using a new neighborhood structure. Technical report, Department of Computer Science, Catholic U. of Rio de Janeiro, Rio de Janeiro, Brazil.

The authors propose a hybridization of GRASP and VND to solve this problem arising in biology. The greedy criterion is based on insertions of new taxons into a branch of the

current partial solution. The local search phase uses a VND strategy and a new neighborhood structure, called 2-SPR and based on the iterative removing of branches on the current solution. See also page 45.

Souza, M., N. Maculan, and L. Ochi (2003). A GRASP-tabu search algorithm for school timetabling problems. In M. Resende and J. de Sousa (Eds.), *Metaheuristics: Computer decision-making*, pp. 659–672. Kluwer Academic Publishers.

A hybrid approach is proposed that uses a greedy randomized construction phase to obtain a feasible solution to be hopefully improved with tabu search. See also page 33.

5. SOFTWARE

This section lists papers that describe computer software for implementing the GRASP framework, as well as codes for solving specific problems.

Andreatta, A., S. Carvalho, and C. Ribeiro (1998). An object-oriented framework for local search heuristics. In *Proceedings of the 26th TOOLS USA '98 – Technology of Object-Oriented Languages and Systems*, pp. 33–45. IEEE Computer Society.

A unified object-oriented framework is proposed for systematically comparing strategies and parameters of different heuristics.

Andreatta, A., S. Carvalho, and C. Ribeiro (2002). A framework for the development of local search heuristics for combinatorial optimization problems. In S. Voss and D. Woodruff (Eds.), *Optimization Software Class Libraries*, pp. 59–79. Kluwer Academic Publishers.

An architectural basis both for the implementation and for the comparison of different local search heuristics is given. Through the use of abstract classes, the framework encapsulates different aspects usually involved in local searches, such as methods for constructing an initial feasible solution, for generating a suitable neighborhood, and for choosing the suitable movement selection criteria.

Festa, P., P. Pardalos, and M. Resende (2001). Algorithm 815: FORTRAN subroutines for computing approximate solution to feedback set problems using GRASP. *ACM Transactions on Mathematical Software* **27**, 456–464.

A set of ANSI standard Fortran 77 subroutines for approximately solving the feedback vertex and arc set problems is described.

Pardalos, P., L. Pitsoulis, and M. Resende (1997). Algorithm 769: Fortran subroutines for approximate solution of sparse quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software* **23**, 196–208.

A version of the GRASP for the quadratic assignment problem of Li, Pardalos, and Resende (1994), tailored for sparse instances is proposed. A set of ANSI standard Fortran 77 subroutines is described. See also page 21.

Resende, M., T. Feo, and S. Smith (1998). Algorithm 787: Fortran subroutines for approximate solution of maximum independent set problems using GRASP. *ACM Transactions on Mathematical Software* **24**, 386–394.

A set of ANSI standard Fortran 77 subroutines for the maximum independent set problem is described. The GRASP used to produce the solutions is proposed in Feo, Resende, and Smith (1994). See also page 17.

Resende, M., P. Pardalos, and Y. Li (1996). Algorithm 754: Fortran subroutines for approximate solution of dense quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software* **22**, 104–118.

A set of ANSI standard Fortran 77 subroutines is proposed for dense quadratic assignment problems having at least one symmetric flow or distance matrix. This is an optimized implementation of the algorithm described in Li, Pardalos, and Resende (1994). See also page 22.

Resende, M., L. Pitsoulis, and P. Pardalos (2000). Fortran subroutines for computing approximate solutions of MAX-SAT problems using GRASP. *Discrete Applied Mathematics* **100**, 95–113.

A set of Fortran subroutines for MAX-SAT problems is described. The algorithm implemented was proposed in Resende, Pitsoulis, and Pardalos (1997). See also page 34.

Ribeiro, C. and M. Resende (1999). Algorithm 797: Fortran subroutines for approximate solution of graph planarization problems using GRASP. *ACM Transactions on Mathematical Software* **25**, 341–352.

A set of Fortran subroutines are described, that implements the GRASP for graph planarization of Resende and Ribeiro (1997). See also page 18 and page 47.

6. PARALLEL GRASP

GRASP can be easily implemented in parallel. In fact, linear speedup can be expected from a straightforward implementation on independent processors. The following papers deal with parallel GRASP.

Aiex, R. (2002). *Uma investigação experimental da distribuição de probabilidade de tempo de solução em heurísticas GRASP e sua aplicação na análise de implementações paralelas*. Ph. D. thesis, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil.

This thesis describes a new methodology for the analysis of GRASP. Hybrid strategies with path-relinking are also proposed. These are studied on the 3-index assignment problem as well as the job shop scheduling problem and analyzed with the proposed methodology to predict qualitatively how the quality of the solution varies in a parallel independent GRASP. See also page 18 and page 30.

Aiex, R., S. Binato, and M. Resende (2003). Parallel GRASP with path-relinking for job shop scheduling. *Parallel Computing* **29**, 393–430.

In this paper, a parallel GRASP with path-relinking as intensification strategy for the job shop problem is described using some ideas proposed in the GRASP of Binato et al. (2002). While in Binato et al. (2002) the greedy function value of an operation k is the makespan resulting from the inclusion of k to the already scheduled operations, in this paper the authors propose the so called *time-remaining* function. It is a greedy function to be used in conjunction with the makespan that favors operations from jobs having long remaining processing times. Two parallelization strategies are proposed: an independent and a cooperative strategy. The independent strategy shows a sub-linear speedup, while the cooperative one shows an approximate linear speedup, thus confirming that the extra time spent for processes communication is compensated by an increase in solution quality. See also page 6 and page 30.

Aiex, R. and M. Resende (2003). Parallel strategies for GRASP with path-relinking. Technical report, Internet and Network Systems Research Center, AT&T Labs Research, Florham Park, NJ.

Independent and cooperative parallel strategies are described and implemented for the 3-index assignment problem and the job-shop scheduling problem. The computational results for independent parallel strategies are shown to qualitatively behave as predicted by the criterion. See also page 7.

Aiex, R., M. Resende, and C. Ribeiro (2002). Probability distribution of solution time in GRASP: An experimental investigation. *Journal of Heuristics* **8**, 343–373.

The authors study the probability distributions of solution time to a sub-optimal target value in five GRASPs that have appeared in the literature and for which source code

is available. The distributions are estimated by running 12,000 independent runs of the heuristic. Standard methodology for graphical analysis is used to compare the empirical and theoretical distributions and estimate the parameters of the distributions. They conclude that the solution time to a sub-optimal target value fits a two-parameter exponential distribution. Hence, it is possible to approximately achieve linear speed-up by implementing GRASP in parallel.

Alvim, A. (1998). Parallelization strategies for the metaheuristic GRASP. Master's thesis, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, RJ 22453-900 Brazil.

Two parallelization strategies for GRASP are discussed and compared: parallelization by distributing GRASP iterations and parallelization by varying the GRASP random parameter α . Both strategies are adapted to several parallel computation models, such as MPI (Message Passing Interface) and PVM (Parallel Virtual Machine). In Portuguese.

Alvim, A. and C. Ribeiro (1998). Load balancing in the parallelization of the metaheuristic GRASP. Technical report, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, RJ 22453-900 Brazil.

Two parallelization strategies for GRASP are compared. The difference between the two strategies concerns the way in which data is partitioned: pre-scheduled (static load balancing) or self-scheduled (dynamic load balancing). The strategies have been tested considering an application to optimal traffic assignment in TDMA satellite system. Best results have been obtained by using the self-scheduling strategy. In Portuguese.

Catalano, M. F. and F. Malucelli (2000). Parallel randomized heuristics for the set covering problem. Technical report, Transportation and traffic engineering section, Delft U. of Technology, 2600 AG Delft, The Netherlands. To appear in International J. of Computer Research.

Parallel heuristics for set covering are presented. These include a GRASP.

Diaz, J., J. Velázquez, and J. Solis (1999). Un modelo tipo GRASP para la paralelización de algoritmos en computadoras MIMD: Aplicación al algoritmo de Jarmenson. Technical report, Universidad Autónoma del Estado de Morelos, Facultad de Ciencias, Cuernavaca, Morelos, Mexico.

In this paper, the authors use a GRASP to optimize operations needed to parallelize algorithms on a MIMD. The greedy criterion is to minimize the cost associated with communication among processors, while local search uses an exchange neighborhood structure. In Spanish.

Duni, S., P. Pardalos, and M. Resende (2002). Parallel metaheuristics for combinatorial optimization. In R. Corrêa, I. Dutra, M. Fiallos, and F. Gomes (Eds.), *Models for Parallel and Distributed Computation – Theory, Algorithmic Techniques and Applications*, pp. 179–206. Kluwer Academic Publishers.

A survey of parallel metaheuristics including GRASP.

Feo, T., M. Resende, and S. Smith (1994). A greedy randomized adaptive search procedure for maximum independent set. *Operations Research* **42**, 860–878.

A GRASP for approximately solving the maximum independent set problem is described. The proposed heuristic can be easily implemented in parallel by decomposing the problem into smaller subproblems, each defined by conditioning on vertices being in the solution. An implementation of this algorithm was tested on a MIMD computer with up to eight processors. Average linear speedup is observed. See also page 15.

Martins, S. (1999). *Estratégias de Paralelização de Metaheurísticas em Ambientes de Memória Distribuída*. Ph. D. thesis, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil.

This thesis considers parallelization strategies for metaheuristics in distributed memory environments. GRASPs for the Steiner tree problem in graphs are described and implemented in parallel. In Portuguese.

- Martins, S., M. Resende, C. Ribeiro, and P. Pardalos (2000). A parallel GRASP for the Steiner tree problem in graphs using a hybrid local search strategy. *Journal of Global Optimization* **17**, 267–283.
A hybrid parallel GRASP for the Steiner problem in graphs is described. See also page 17.
- Martins, S., C. Ribeiro, and M. Souza (1998). A parallel GRASP for the Steiner problem in graphs. In A. Ferreira and J. Rolim (Eds.), *Proceedings of IRREGULAR'98 – 5th International Symposium on Solving Irregularly Structured Problems in Parallel*, Volume 1457 of *Lecture Notes in Computer Science*, pp. 285–297. Springer-Verlag.
A parallelization of a sequential GRASP for the Steiner minimal tree problem is proposed. See also page 17.
- Murphey, R., P. Pardalos, and L. Pitsoulis (1998). A parallel GRASP for the data association multidimensional assignment problem. In P. Pardalos (Ed.), *Parallel Processing of Discrete Problems*, Volume 106 of *The IMA Volumes in Mathematics and its Applications*, pp. 159–180. Springer-Verlag.
A GRASP for finding good solutions for the data association multidimensional assignment problem is described. The proposed method can be easily parallelized to substantially decrease the running time. See also page 20.
- Pardalos, P., L. Pitsoulis, T. Mavridou, and M. Resende (1995). Parallel search for combinatorial optimization: Genetic algorithms, simulated annealing and GRASP. In A. Ferreira and J. Rolim (Eds.), *Parallel Algorithms for Irregularly Structured Problems, Proceedings of the Second International Workshop – Irregular'95*, Volume 980 of *Lecture Notes in Computer Science*, pp. 317–331. Springer-Verlag.
Parallel search techniques for approximating the global optimal solution of combinatorial optimization problems are addressed. For large-scale problems, one of the main limitations of heuristic search is its computational complexity. Efficient parallel implementation of search algorithms can significantly increase the size of the problems that can be solved.
- Pardalos, P., L. Pitsoulis, and M. Resende (1995). A parallel GRASP implementation for the quadratic assignment problem. In A. Ferreira and J. Rolim (Eds.), *Parallel Algorithms for Irregularly Structured Problems – Irregular'94*, pp. 115–130. Kluwer Academic Publishers.
Efficient parallel techniques for large-scale sparse quadratic assignment problems are discussed. The GRASP iterations are distributed among the processors. Each processor is given its own input data and random number sequence and are run independently. A shared global variable stores the value of the incumbent solution. see also page 20.
- Pardalos, P., L. Pitsoulis, and M. Resende (1996). A parallel GRASP for MAX-SAT problems. *Lecture Notes in Computer Science* **1184**, 575–585.
A parallel GRASP for weighted maximum satisfiability (MAX-SAT) problem is proposed. The parallel implementation distributes the GRASP iterations among several processors operating in parallel, avoiding that two processors have as input the same random number generator seed. The best solution found among all processors is identified and used as solution of the problem. See also page 34.
- Ribeiro, C. and I. Rosseti (2002). A parallel GRASP for the 2-path network design problem. *Lecture Notes in Computer Science* **2004**, 922–926.
A parallel GRASP with path-relinking is proposed for solving 2-path network design problem. See also page 18 and page 42.
- Rivera, L. (1998). Evaluation of parallel implementations of heuristics for the course scheduling problem. Master's thesis, Instituto Tecnológico y de Estudios Superiores de Monterrey, Monterrey, Mexico.
This thesis presents several parallel implementations of heuristics for the course scheduling problem. One of the heuristics is a GRASP. In Spanish. See also page 33.

7. GRAPH THEORETICAL APPLICATIONS

Perhaps the type of problems where GRASP has been most applied to are problems dealing with graph theory. Below are papers on graph theoretical applications.

Abello, J., P. Pardalos, and M. Resende (1999). On maximum clique problems in very large graphs. In J. Abello and J. Vitter (Eds.), *External memory algorithms and visualization*, Volume 50 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pp. 119–130. American Mathematical Society.

An approach for clique and quasi-clique computations in very large multi-digraphs is presented. The authors discuss graph decomposition schemes that break up the original problem into several pieces of manageable dimensions. A semi-external memory GRASP is presented to approximately solve the maximum clique problem and maximum quasi-clique problem. See also page 38.

Abello, J., M. Resende, and S. Sudarsky (2002). Massive quasi-clique detection. *Lecture Notes in Computer Science* **2286**, 598–612.

The problem of detecting dense subgraphs (quasi-cliques) in massive sparse graphs whose vertex set fits in RAM is addressed. A GRASP is designed for extracting dense subgraphs in both nonbipartite and bipartite graphs. See also page 38.

Ahuja, R., J. Orlin, and D. Sharma (2001). Multi-exchange neighborhood search structures for the capacitated minimum spanning tree problem. *Mathematical Programming* **91**, 71–97.

Two very large scale neighborhood search algorithms are proposed for the capacitated minimum spanning tree problem. The first one uses a node based neighborhood structure, defined by performing multi-exchanges involving several trees, where each tree contributes a subtree. The second uses a tree based neighborhood structure, obtained by allowing multi-exchanges, where each tree contributes a subtree. The search algorithms are guided by a GRASP.

Ahuja, R., J. Orlin, and D. Sharma (2003). A composite very large-scale neighborhood structure for the capacitated minimum spanning tree problem. *Operations Research Letters* **31**(3), 185–194.

The authors propose a composite neighborhood structure that combines both the node based and the tree based neighborhoods proposed in Ahuja et al. (2001). They compare their algorithm with the best state-of-art methods, including a GRASP.

Arráziz, E., A. Martínez, O. Meza, and M. Ortega (2001). GRASP and tabu search algorithms for computing the forwarding index in a graph. In *Proceedings of MIC'2001*, pp. 367–370.

The forwarding index in a graph is a measure of the load or of the congestion of vertices once the paths between vertices have been computed. To efficiently compute this number, a GRASP and tabu search is proposed. The GRASP greedy choice considers the length of paths connecting nonadjacent nodes, while two different local search strategies are used. The first replaces a path passing through an internal node with maximum load with another path joining the same endpoints. The second replaces a path of maximum length.

Boeres, M., C. Ribeiro, and I. Bloch (2003). Randomized algorithms for scene recognition by inexact graph matching. Technical report, Computer Science Department, Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil. To appear in *Lecture Notes in Computer Science*.

The algorithm proposed to solve a scene recognition problem consists of a randomized construction procedure and a local search procedure. In the construction procedure, the greedy function has two terms representing, respectively, the node and edge contributions to the measure of the solution quality associated with the correspondence. Given a feasible solution x , the neighborhood structure used during local search considers as neighbors of x all feasible solutions that can be obtained by changing some association.

Canuto, S., M. Resende, and C. Ribeiro (2001). Local search with perturbation for the prize-collecting Steiner tree problems in graphs. *Networks* **38**, 50–58.

Several local search strategies for the prize collecting Steiner tree problem in graphs are investigated, including path relinking, VNS, and a GRASP with cost perturbations. See also page 5, page 7, and page 39.

Corberán, A., R. Martí, and J. Sanchís (2002). A GRASP heuristic for the mixed Chinese postman problem. *European Journal of Operational Research* **142**, 70–80.

A GRASP for the mixed Chinese postman problem is given. On page 29 details of the implementation of proposed GRASP are given.

de Souza, M., C. Duhamel, and C. Ribeiro (2003). A GRASP heuristic for the capacitated minimum spanning tree problem using a memory-based local search strategy. In M. Resende and J. de Sousa (Eds.), *Metaheuristics: Computer decision-making*, pp. 627–658. Kluwer Academic Publishers.

A GRASP for the capacitated minimum spanning tree problem is proposed. It uses a novel local search that considers a new path-based neighborhood structure, defined by path exchanges. The GRASP also makes use of some short term memory elements of tabu search and that implements a randomized version of a savings heuristic in the construction phase. A post-optimization phase is realized by applying a path-relinking procedure. See also page 8.

Feo, T., M. Resende, and S. Smith (1994). A greedy randomized adaptive search procedure for maximum independent set. *Operations Research* **42**, 860–878.

A GRASP for maximum independent set is described. The greedy function chosen orders admissible vertices with respect to the minimum admissible vertex degree. The adjective admissible is referred to a vertex that is not adjacent to any vertex in the current independent set. The neighborhood definition used in the local search is $(2, 1)$ -exchange, where two nonadjacent vertices can be added to the current solution if a single vertex from the solution is removed. The proposed heuristic can be easily implemented in parallel by decomposing the problem into smaller subproblems, each defined by conditioning on vertices being in the solution. See also page 12.

Festa, P., P. Pardalos, M. Resende, and C. Ribeiro (2002). Randomized heuristics for the MAX-CUT problem. *Optimization Methods and Software* **7**, 1033–1058.

Randomized heuristics for the MAX-CUT problem are proposed and tested. These include a GRASP and hybrids. See also page 8.

Holmqvist, K., A. Migdalas, and P. Pardalos (1998). A GRASP algorithm for the single source uncapacitated minimum concave-cost network flow problem. In P. Pardalos and D.-Z. Du (Eds.), *Network design: Connectivity and facilities location*, Volume 40 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pp. 131–142. American Mathematical Society.

The single source uncapacitated version of the minimum concave-cost network flow problem is considered. It requires establishing a minimum cost flow through a given network from a single source to a set of sinks. The authors propose a GRASP that can be trivially implemented on parallel processors. The construction phase iteratively builds a tree starting from the source node. The elements of the restricted candidate list are end nodes of arcs with a cost close to the best one. The local search phase applies either of the two local search variants proposed by Guisewite and Pardalos (1990).

Laguna, M. and R. Martí (2001). A GRASP for coloring sparse graphs. *Computational Optimization and Applications* **19**, 165–178.

A GRASP for graph coloring is presented. The GRASP construction phase constructs the next coloring, one color at a time. The greedy function chooses the vertex having the maximum degree among the uncolored vertices adjacent to at least one colored vertex.

At each step, the local search combines the two smallest cardinality color classes into one and tries to find a valid color for each violating vertex.

Macambira, E. and C. de Souza (1997a). The edge-weighted clique problem: Valid inequalities, facets and polyhedral computations. Technical Report IC-97-14, Instituto de Computação, Universidade Estadual de Campinas, Campinas, Brazil.

The facial structure of the polytope associated with the edge-weighted clique problem is studied and new classes of facets are introduced. To obtain initial lower bounds, a GRASP is used.

Macambira, E. and C. de Souza (1997b). A GRASP for the maximum clique problem with weighted edges. In *Proceedings of the XXIX Brazilian Symposium on Operations Research*, pp. 70.

A branch-and-cut algorithm is proposed for the maximum clique problem with weighted edges. The initialization phase of the algorithm uses a GRASP to generate good starting solutions. The greedy function minimizes the sum of weights of the edges outgoing from vertices in the clique. The local search uses the exchange of one element in the current clique with one not in it.

Macambira, E. and C. Meneses (1998). A GRASP algorithm for the maximum weighted edge subgraph problem. Technical report, Department of Statistics and Computation, University of Ceará, Fortaleza, CE 60740-000 Brazil.

GRASP for the maximum weighted edge subgraph problem is proposed. The greedy function of the construction phase favors the vertices corresponding to the maximum sum of the weights associated with its outgoing edges. The local search tries to improve the actual solution by simply swapping one element in the solution set with one not belonging to the solution. In Portuguese.

Marinakis, Y. and A. Migdalas (2003). Expanding neighborhood GRASP for the Traveling Salesman Problem. Technical report, Technical University of Crete, Chania 73100, Greece.

A GRASP is proposed for the traveling salesman problem using a VNS-like local search. See also page 8 and page 30.

Martí, R. (2001). Arc crossing minimization in graphs with GRASP. *IIE Transactions* **33**, 913–919.

A GRASP for minimizing straight-line crossings in hierarchical graphs is presented. GRASP is shown to be faster than more complex heuristics but produces lower-quality solutions. Though it is not as fast as simple heuristics, it finds better-quality solutions.

Martí, R. and V. Estruch (2001). Incremental bipartite drawing problem. *Computers and Operations Research* **28**, 1287–1298.

The goal of limiting the number of arc crossings is a well accepted criterion of how well a graph is drawn. Incremental graph drawing supports interactive updates by users. A GRASP is proposed for the incremental arc crossing minimization problem for bipartite graphs. See page 46.

Martí, R. and M. Laguna (2003). Heuristics and meta-heuristics for 2-layer straight line crossing minimization. *Discrete Applied Mathematics* **127**, 665–678.

Extensive computational results are presented using 12 heuristics and two metaheuristics for the 2-layer straight line crossing minimization problem. One of the metaheuristics is a GRASP. See page 46.

Martins, S., P. Pardalos, M. Resende, and C. Ribeiro (1999). Greedy randomized adaptive search procedures for the Steiner problem in graphs. In P. Pardalos, S. Rajasekaran, and J. Rolim (Eds.), *Randomization methods in algorithmic design*, Volume 43 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pp. 133–145. American Mathematical Society.

Four versions of a GRASP for approximately solving general instances of the Steiner problem in graphs are proposed. One is implemented and tested. The construction phase

is based on the distance network heuristic. The local search is based on insertions and eliminations of nodes to and from the current solution.

Martins, S., M. Resende, C. Ribeiro, and P. Pardalos (2000). A parallel GRASP for the Steiner tree problem in graphs using a hybrid local search strategy. *Journal of Global Optimization* **17**, 267–283.

A GRASP for the Steiner problem in graphs is described. See page 13.

Martins, S., C. Ribeiro, and M. Souza (1998). A parallel GRASP for the Steiner problem in graphs. In A. Ferreira and J. Rolim (Eds.), *Proceedings of IRREGULAR'98 – 5th International Symposium on Solving Irregularly Structured Problems in Parallel*, Volume 1457 of *Lecture Notes in Computer Science*, pp. 285–297. Springer-Verlag.

A parallelization of a sequential GRASP for the Steiner minimal tree problem is proposed. The procedure implemented is one of the procedures described in Martins, Pardalos, Resende, and Ribeiro (1999).

Motta, L., L. Ochi, and C. Martinhon (2001). GRASP metaheuristics to the generalized covering tour problem. In *Proceedings of MIC'2001*, pp. 387–391.

A GRASP for the generalized covering tour problem is presented.

Osman, I., B. Al-Ayoubi, and M. Barake (2003). A greedy random adaptive search procedure for the maximal planar graph problem. *Computers and Industrial Engineering* **45**, 635–651.

A GRASP is proposed and tested for the weighted maximal planar graph problem.

GRASP implementation details are given on page 46.

Osman, I., B. Al-Ayoubi, M. Barake, and M. Hasan (2000). A greedy random adaptive search procedure for the weighted maximal planar graph problem. Technical report, School of Business and Center for Advanced Mathematical Sciences, American University of Beirut, Beirut, Lebanon.

A GRASP is proposed and tested for the weighted maximal planar graph problem.

Details of the GRASP implementation are given on page 46.

Osman, I., M. Hasan, and A. Abdullah (2002). Linear programming based meta-heuristics for the weighted maximal planar graph. *Journal of the Operational Research Society* **53**, 1142–1149.

Two meta-heuristics are described, including a GRASP, for the weighted maximal planar graph problem. Both are derived from an ILP relaxation. See page 46.

Pardalos, P., T. Qian, and M. Resende (1999). A greedy randomized adaptive search procedure for the feedback vertex set problem. *Journal of Combinatorial Optimization* **2**, 399–412.

A GRASP is described for the feedback vertex set problem on a digraph. Several greedy functions are tested, all of them taking into account vertices with high degree. The local search procedure tries at each iteration to eliminate redundant vertices. Some efficient problem reduction techniques are also described. They are useful both to simplify the problem instance and to determine whether a digraph is acyclic.

Pardalos, P., M. Resende, and J. Rappe (1998). An exact parallel algorithm for the maximum clique problem. In R. D. Leone, A. Murli, P. Pardalos, and G. Toraldo (Eds.), *High performance algorithms and software in nonlinear optimization*, pp. 279–300. Kluwer Academic Publishers.

A parallelized version of the exact algorithm of Carraghan and Pardalos (1990) for the unweighted maximum clique problem is described. A variant of the GRASP for the maximum independent set problem of Feo, Resende, and Smith (1994) is used for computing feasible solutions.

Resende, M., T. Feo, and S. Smith (1998). Algorithm 787: Fortran subroutines for approximate solution of maximum independent set problems using GRASP. *ACM Transactions on Mathematical Software* **24**, 386–394.

A set of ANSI standard Fortran 77 subroutines to find an approximate solution of a maximum independent set problem using the GRASP of Feo, Resende, and Smith (1994). See page 10.

- Resende, M. and C. Ribeiro (1997). A GRASP for graph planarization. *Networks* **29**, 173–189.
 A GRASP for the graph planarization problem is described in detail, extending the two-phase heuristic of Goldschmidt and Takvorian (*Networks*, v. 24, pp. 69–73, 1994). See also page 46.
- Ribeiro, C. and M. Resende (1999). Algorithm 797: Fortran subroutines for approximate solution of graph planarization problems using GRASP. *ACM Transactions on Mathematical Software* **25**, 341–352.
 A set of Fortran subroutines that implements the GRASP for graph planarization of Resende and Ribeiro (1997) is described. See also page 11 and page 47.
- Ribeiro, C. and I. Rosseti (2002). A parallel GRASP for the 2-path network design problem. *Lecture Notes in Computer Science* **2004**, 922–926.
 A parallel GRASP with path-relinking is proposed for finding approximate solutions to the 2-path network design problem. See also page 13 and page 42.
- Ribeiro, C., E. Uchoa, and R. Werneck (2002). A hybrid GRASP with perturbations for the Steiner problem in graphs. *INFORMS Journal on Computing* **14**, 228–246.
 In this hybrid GRASP, an adaptive path-relinking procedure is used as a post-optimization strategy. The GRASP construction phase is replaced by either one of several different construction procedures that apply weight perturbation strategies combining intensification and diversification elements. The local search phase circularly explores two different strategies. The first defines a simple node-based neighborhood structure. The second one uses a key-path-based neighborhood, where a key-node is a Steiner node with degree at least three and a key-path is a Steiner tree T whose extremities are either terminals of key-node (if there are any, intermediate nodes are Steiner node with degree two in T). See also page 6 and page 9.
- Rosseti, I. (2003). *Heurísticas para o problema de síntese de redes a 2-caminhos*. Ph. D. thesis, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil.
 This thesis proposes sequential and parallel heuristics for the 2-path network design problem. These include variants and combinations of GRASP. In Portuguese. See also page 42.

8. QUADRATIC AND OTHER ASSIGNMENT PROBLEMS

GRASP has been applied to the quadratic assignment problems, as well as other assignment problems, such as biquadratic, three index, and multidimensional assignment. The following papers fall into this category.

- Ahuja, R., J. Orlin, and A. Tiwari (2000). A greedy genetic algorithm for the quadratic assignment problem. *Computers and Operations Research* **27**, 917–934.
 A genetic algorithm for the QAP that incorporates the construction phase of the GRASP for QAP of Li, Pardalos, and Resende (1994) to generate the initial population is described. See also page 6.
- Aiex, R. (2002). *Uma investigação experimental da distribuição de probabilidade de tempo de solução em heurísticas GRASP e sua aplicação na análise de implementações paralelas*. Ph. D. thesis, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil.
 This thesis describes a new methodology for the analysis of GRASP. Hybrid strategies with path-relinking are also proposed. These are studied on the 3-index assignment problem as well as the job shop scheduling problem. See also page 11 and page 30.
- Aiex, R., M. Resende, P. Pardalos, and G. Toraldo (2000). GRASP with path relinking for the three-index assignment problem. Technical report, AT&T Labs Research, Florham Park, NJ 07733. To appear in *INFORMS J. on Computing*.

Variants of GRASP with path relinking for the three index assignment problem (AP3) are presented. Computational results show clearly that this GRASP for AP3 benefits from path relinking and that the variants considered in this paper compare well with previously proposed heuristics for this problem. The authors also show that the random variable time to target solution, for all proposed GRASP with path relinking variants, fits a two-parameter exponential distribution. To illustrate the consequence of this, one of the variants of GRASP with path relinking is shown to benefit from parallelization. See also page 7.

Bard, J. (1997). An analysis of a rail car unloading area for a consumer products manufacturer. *Journal of the Operational Research Society* **48**, 873–883.

This paper discusses how to design and analyze the rail-car unloading area of Procter & Gamble's principal laundry detergent plant. The related combinatorial problem of assigning rail-cars to positions on the platform and unloading equipment to rail-cars is modeled as a mixed-integer nonlinear program. See also page 36.

Feo, T. and J. González-Velarde (1995). The intermodal trailer assignment problem: Models, algorithms, and heuristics. *Transportation Science* **29**, 330–341.

The problem of optimally assigning highway trailers to rail-car hitches in intermodal transportation is addressed. Using a set covering formulation, the problem is modeled as an integer linear program, whose linear programming relaxation yields a tight lower bound. This formulation also provides the basis for developing a branch-and-bound algorithm and a GRASP for solving the problem. See also page 37.

Fleurent, C. and F. Glover (1999). Improved constructive multistart strategies for the quadratic assignment problem using adaptive memory. *INFORMS Journal on Computing* **11**, 198–204.

This paper illustrates that constructive multistart methods, such as Random Restart and GRASP, can be improved by the addition of memory and associated heuristic search principles. It shows that the GRASP for QAP of Li, Pardalos, and Resende (1994) can be improved upon by using these memory strategies. See also page 5.

Li, Y., P. Pardalos, and M. Resende (1994). A greedy randomized adaptive search procedure for the quadratic assignment problem. In P. Pardalos and H. Wolkowicz (Eds.), *Quadratic assignment and related problems*, Volume 16 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pp. 237–261. American Mathematical Society.

A GRASP for the quadratic assignment problem is described. Construction first makes two assignments, and then completes the solution by making assignments, one at a time. The greedy function is assignment interaction cost. The local search procedure is a 2 assignment exchange.

Liu, X., P. Pardalos, S. Rajasekaran, and M. Resende (2000). A GRASP for frequency assignment in mobile radio networks. In S. Rajasekaran, P. Pardalos, and F.Hsu (Eds.), *Mobile Networks and Computing*, Volume 52 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pp. 195–201. American Mathematical Society.

A GRASP for frequency assignment is described. Local search uses simulated annealing. See also page 40.

Lourenço, H. R. and D. Serra (1998). Adaptive approach heuristics for the generalized assignment problem. Technical Report 288, Department of Economics and Business, Universitat Pompeu Fabra, Barcelona, Spain.

Several heuristics based a hybrid approach are proposed. This includes a GRASP, whose greedy choice is to prefer tasks using small amounts of agent resource. In the improvement phase, several local searches are implemented, among them a descent local search and a search based on ejection chain neighborhood.

Mavridou, T., P. Pardalos, L. Pitsoulis, and M. Resende (1998). A GRASP for the biquadratic assignment problem. *European Journal of Operational Research* **105**, 613–621.

A GRASP for the biquadratic assignment problem is proposed. The construction phase has two stages. The first stage simultaneously makes four assignments, selecting the pairs corresponding to the smallest interaction costs, while the second stage makes the remaining assignments, one at a time. The greedy function in the second stage selects the assignment corresponding to the minimum interaction cost with respect to the already-made assignments. In the local search phase, 2-exchange local search is applied to the permutation constructed in the first phase.

Murphey, R., P. Pardalos, and L. Pitsoulis (1998a). A greedy randomized adaptive search procedure for the multitarget multisensor tracking problem. In P. Pardalos and D.-Z. Du (Eds.), *Network design: Connectivity and facilities location*, Volume 40 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pp. 277–301. American Mathematical Society.

A GRASP is presented for the multitarget multisensor tracking problem, which can be interpreted as a collection of multidimensional assignment problems. A likelihood cost function and partitioning constraint set are developed. The GRASP construction phase creates the restricted candidate list containing the most likely to occur (lower cost) tuples. The local search explores all 2-exchange permutations.

Murphey, R., P. Pardalos, and L. Pitsoulis (1998b). A parallel GRASP for the data association multidimensional assignment problem. In P. Pardalos (Ed.), *Parallel Processing of Discrete Problems*, Volume 106 of *The IMA Volumes in Mathematics and its Applications*, pp. 159–180. Springer-Verlag.

A parallel GRASP for the data association multidimensional assignment problem is described. At each discrete time interval, the data set is formulated as a multidimensional assignment problem (MAP) with a maximum likelihood cost function. A near-optimal solution to each MAP is obtained with a GRASP. See also page 13.

Oliveira, C. and F. Gomes (1999). Two metaheuristics for channel allocation in mobile telephony. Technical report, Artificial Intelligence Laboratory, Universidade Federal do Ceará, Fortaleza, Brazil.

This frequency assignment problem consists in minimizing the total system interference in mobile phone covered areas, with respect to co-channel and adjacent-channel interference. Two metaheuristics are proposed: GRASP and Asynchronous Team (A-Team). See also page 40.

Oliveira, C., P. Pardalos, and M. Resende (2003). GRASP with path-relinking for the QAP. In T. Ibaraki and Y. Yoshitomi (Eds.), *Proceedings of the Fifth Metaheuristics International Conference (MIC2003)*, pp. 57–1 – 57–6.

In this paper, a GRASP using path relinking as intensification phase is proposed for the quadratic assignment problem. The GRASP is based on the GRASP proposed in Li et al. (1994). See also page 8.

Oliveira, C., P. Pardalos, and M. Resende (2004). GRASP with path-relinking the the quadratic assignment problem. *Lecture Notes in Computer Science*. To appear.

This paper describes a GRASP for quadratic assignment problem which uses a faster local search than in Li, Pardalos, and Resende (1994) and has a path-relinking intensification component. Computational results show that GRASP is sped up considerably when used in conjunction with the path-relinking component.

Pardalos, P., L. Pitsoulis, and M. Resende (1995). A parallel GRASP implementation for the quadratic assignment problem. In A. Ferreira and J. Rolim (Eds.), *Parallel Algorithms for Irregularly Structured Problems – Irregular'94*, pp. 115–130. Kluwer Academic Publishers.

Efficient parallel techniques for large-scale sparse quadratic assignment problems are discussed. The paper provides a detailed description of a parallel implementation on

- an MIMD computer of the sequential GRASP proposed by Li, Pardalos, and Resende (1994) for solving the QAP. See also page 13.
- Pardalos, P., L. Pitsoulis, and M. Resende (1997). Algorithm 769: Fortran subroutines for approximate solution of sparse quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software* **23**, 196–208.
- A version of the GRASP for the quadratic assignment problem of Li, Pardalos, and Resende (1994), tailored for sparse instances is proposed. A set of ANSI standard Fortran 77 subroutines are described. See also page 10.
- Pardalos, P., K. Ramakrishnan, M. Resende, and Y. Li (1997). Implementation of a variance reduction based lower bound in a branch and bound algorithm for the quadratic assignment problem. *SIAM Journal on Optimization* **7**, 280–294.
- A branch & bound algorithm for the quadratic assignment problem is proposed. It uses a GRASP to compute upper bounds.
- Pasiliao, E. (1998). A greedy randomized adaptive search procedure for the multi-criteria radio link frequency assignment problem. Technical report, Department of ISE, University of Florida, Gainesville, FL 32611-6595.
- A GRASP for computing approximate solutions to a radio link frequency assignment problem is proposed. The objective is to minimize the order and the span of the solution set. See also page 41.
- Pitsoulis, L. (1999). *Algorithms for nonlinear assignment problems*. Ph. D. thesis, Department of Industrial and Systems Engineering, University of Florida.
- This dissertation presents GRASPs for solving the following NP-hard nonlinear assignment problems (NAPs): quadratic assignment problem (QAP), biquadratic assignment problem (BiQAP), turbine balancing problem (TBP), and multidimensional assignment problem (MAP). Computational results indicate that all of the suggested algorithms are among the best in the literature in terms of solution quality and computational time.
- Pitsoulis, L., P. Pardalos, and D. Hearn (2001). Approximate solutions to the turbine balancing problem. *European J. of Operational Research* **130**, 147–155.
- The turbine balancing problem is formulated as a standard quadratic assignment problem, and a GRASP for solving the resulting problem is presented.
- Prais, M. and C. Ribeiro (2000). Reactive GRASP: An application to a matrix decomposition problem in TDMA traffic assignment. *INFORMS Journal on Computing* **12**, 164–176.
- A GRASP is described for a matrix decomposition problem arising in the context of traffic assignment in communication satellites. A geostationary communication satellite has a number of spot beam antennas covering geographically distributed areas. According to the slot switching configuration on the on-board switch, the uplink traffic received at the satellite has to be immediately sent to ground areas through a set of transponders. The slot switching configurations are determined through the solution of a time slot assignment problem, which is equivalent to the decomposition of a nonnegative traffic matrix into the sum of a family of switching mode matrices. A refinement of GRASP, called Reactive GRASP, is proposed. See also page 5 and page 41.
- Rangel, M., N. Abreu, and P. Boaventura Netto (2000). GRASP para o PQA: Um limite de aceitação para soluções iniciais. *Pesquisa Operacional* **20**, 45–58.
- A modified version of the GRASP proposed in Li, Pardalos, and Resende (1994) for the quadratic assignment problem is presented. It computes a normalized limit cost, defined with the aid of QAP upper and lower bounds easily obtained and discards all solutions with cost less than the computed limit. In Portuguese. See also page 6.

Rangel, M., N. de Abreu, P. Boaventura Netto, and M. Boeres (1998). A modified local search for GRASP in the quadratic assignment problem. Technical report, Production Engineering Program, COPPE, Federal University of Rio de Janeiro, Rio de Janeiro, RJ Brazil.

An improvement of the local search phase of the GRASP proposed by Li, Pardalos, and Resende (1994) for solving the quadratic assignment problem is proposed. The new strategy amplifies the local search range and improves the local search's efficiency.

Resende, M., P. Pardalos, and Y. Li (1996). Algorithm 754: Fortran subroutines for approximate solution of dense quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software* **22**, 104–118.

A set of ANSI standard Fortran 77 subroutines is described for dense quadratic assignment problems having at least one symmetric flow or distance matrix. It is an optimized implementation of the algorithm described in Li, Pardalos, and Resende (1994). See also page 11.

Robertson, A. (2001). A set of greedy randomized adaptive local search procedure (GRASP) implementations for the multidimensional assignment problem. *Computational Optimization and Applications* **19**, 145–164.

Four GRASP implementations are introduced for the multidimensional assignment problem. They combine two constructive methods (randomized greedy and randomized max regret) and two local search methods (2-exchange and variable depth exchange). The greedy function of the randomized max regret construction method is a measure of the competition between the two leading cost candidates. The maximum regret value corresponds to the candidate assignment that has the largest winning margin between itself and its next highest competitor. The variable depth exchange is an extension of the 2-exchange method that allows a more extensive search of the surrounding neighborhood space.

Sosnowska, D. (2000). Optimization of a simplified fleet assignment problem with metaheuristics: Simulated annealing and GRASP. In P. Pardalos (Ed.), *Approximation and complexity in numerical optimization*. Kluwer Academic Publishers.

Two heuristics based on simulated annealing and GRASP are presented for a simplified fleet assignment problem. Both methods are based on swapping parts of sequence of flight legs assigned to an aircraft (rotation cycle) between two randomly chosen aircrafts. See also page 38.

Urban, T., W.-C. Chiang, and R. Russel (2000). The integrated machine allocation and layout problem. *International Journal of Production Research* **38**, 2911–2930.

GRASP is used to solve quadratic assignment sub-problems in a model that aggregates quadratic assignment problems with several network flow problems with side constraints. This model is used to produce machine layouts where machines are not required to be placed in a functional or cellular layout. See also page 26.

Vieira, C. and P. Gondim (2001). Uma nova estratégia para aplicação do GRASP ao problema de alocação de canal. Technical Report 070/DE9/01, Departamento de Engenharia de Sistemas, Instituto Militar de Engenharia, Rio de Janeiro, Brazil.

A special channel allocation problem arising in telecommunications is addressed. A heuristic that uses the GRASP randomized construction idea is proposed. The chosen greedy function measures the difficulty in assigning a frequency. See also page 42.

9. LOCATION AND LAYOUT

Location and layout are another class of problems successfully handled by GRASP. The following papers show how GRASP is used in this context.

Abdinnour-Helm, S. and S. Hadley (2000). Tabu search based heuristics for multi-floor facility layout. *International Journal of Production Research* **38**, 365–383.

A GRASP/Tabu Search is proposed applying a GRASP to find the initial layout and tabu search to refine the layout. Computational tests indicate that GRASP/Tabu Search compares favorably with other heuristics. See also page 6.

Alvarez-Valdes, R., A. Parajón, and J. Tamarit (2001). A computational study of heuristic algorithms for two-dimensional cutting stock problems. In *Proceedings of MIC'2001*, pp. 7–11.

To solve the column generation subproblem arising while solving two-dimensional cutting stock problem, a Gilmore-Gomory approach and several heuristics, including a GRASP, are proposed. The greedy criterion is the potential benefit of an item, while at each iteration of the local search, waste rectangles produced in the construction phase are merged if possible with adjacent pieces, creating new rectangles that can be cut with greater value.

Alvarez-Valdés, R., A. Parajón, and J. Tamarit (2002). A tabu search algorithm for large-scale guillotine (un)constrained two-dimensional cutting problems. *Computers & Operations Research* **29**, 925–947.

Several heuristics and hybridizations are proposed for the two-dimensional cutting problem, in which a single stock sheet has to be cut into a set of small pieces, while maximizing the value of the pieces cut. See page 7.

Amaldi, E., A. Capone, and F. Malucelli (2003). Planning UMTS base station location: Optimization models with power control and algorithms. *IEEE Transactions on Wireless Communications* **2**(5), 939–952.

Two greedy randomized algorithms and a tabu search are proposed for the UMTS base station location problem. See also page 38.

Amaldi, E., A. Capone, F. Malucelli, and F. Signori (2003). Optimization models and algorithms for downlink UMTS radio planning. In *Wireless Communications and Networking, 2003 (WCNC 2003)*, Volume 2, pp. 827–831.

This paper deals with the UMTS base station location problem. Two mathematical programming formulations are provided for locating directive base stations considering downlink (base station to mobile) direction and assuming a power-based as well or a SIR-based (Signal-to-Interface Ratio) power control mechanism. A GRASP is used to solve the problem. See also page 39.

Areibi, S. (1999). GRASP: An effective constructive technique for VLSI circuit partitioning. In *Proc. IEEE Canadian Conference on Electrical & Computer Engineering (CCECE'99)*, Volume 1, pp. 462–467.

A GRASP is proposed for producing good initial solutions for an iterative improvement technique. At each iteration of the randomized approach, the gains associated with moving modules to the current block being filled are examined, and a restricted candidate list is built using the modules with the highest gains. See also page 26 and page 45.

Areibi, S. and A. Vannelli (1997). A GRASP clustering technique for circuit partitioning. In J. Gu and P. Pardalos (Eds.), *Satisfiability problems*, Volume 35 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pp. 711–724. American Mathematical Society.

A basic node interchange scheme is proposed for solving the circuit partitioning problem. A clustering technique that uses GRASP to generate clusters of moderate sizes is described. Details of the GRASP implementation are given on page 45.

Areibi, S. and A. Vannelli (2000). Efficient hybrid search techniques for circuit partitioning. In *IEEE 4th World Multiconference on Circuits, Systems, Communications & Computers*.

Simulated annealing, tabu search, GRASP, and a genetic algorithm are proposed for circuit partitioning. Two further search techniques are also proposed as hybrids, where a GRASP and a genetic algorithm are used for generating good initial partitions. See also page 7, page 26, and page 45.

- Bautista, J. and J. Pereira (2003). Procedimientos para la localización de áreas de aportación de residuos urbanos. In *27 Congreso Nacional de Estadística e Investigación Operativa*, Lleida, Spain.
 An exact method, a genetic algorithm, and a GRASP are proposed to solve the problem of locating areas where to collect waste products. The two metaheuristics share the same local search strategy that simply looks for redundant elements to be removed from the solution. In Spanish.
- Cano, J., O. Cordón, F. Herrera, and L. Sánchez (2002). A GRASP algorithm for clustering. In F. J. Garijo, J. Santos, and M. Toro (Eds.), *Advances in Artificial Intelligence - IBERAMIA 2002, 8th Ibero-American Conference on AI, Seville, Spain, November 12-15, 2002, Proceedings*, Volume 2527 of *Lecture Notes in Computer Science*, pp. 214–223. Springer.
 A GRASP for cluster analysis is described. See also page 7 and page 27.
- Chiang, W., P. Kouvelis, and T. Urban (2002). Incorporating workflow interference in facility layout design: The quartic assignment problem. *Management Science* **48**(4), 584–590.
 A branch & bound and a tabu search are proposed for the quartic assignment problem. Computational efficiency and performance of the proposed methods are investigated on a set of randomly generated instances by comparing them with the GRASP for the bi-quadratic assignment problem proposed in Mavridou et al. (1998).
- Delmaire, H., J. Díaz, E. Fernández, and M. Ortega (1997). Comparing new heuristics for the pure integer capacitated plant location problem. Technical Report DR97/10, Department of Statistics and Operations Research, Universitat Politècnica de Catalunya, Barcelona, Spain.
 Several heuristics are proposed to solve the pure integer capacitated plant location problem: evolutionary algorithms, GRASP, simulated annealing, and tabu search. All the algorithms share the same neighborhood definition.
- Delmaire, H., J. Díaz, E. Fernández, and M. Ortega (1999). Reactive GRASP and tabu search based heuristics for the single source capacitated plant location problem. *INFOR* **37**, 194–225.
 A reactive GRASP is embedded in a tabu search algorithm for the single source capacitated plant location problem. See also page 8.
- García, J. D. (2001). *Algorithmic approaches for the single source capacitated plant location problem*. Ph. D. thesis, Departamento d'Estadística i Investigació Operativa, Universitat Politècnica de Catalunya, Barcelona, Spain.
 This thesis proposes several algorithmic alternatives based on both exact and approximate methods for efficiently solving the single source capacitated plant location problem. One of the proposals is a reactive GRASP that uses a greedy function based on a percentage of the sum of the cost associated with opening a plant and the cost of allocating clients. The local search procedure uses two neighborhood structures: a client shift neighborhood and a client swap neighborhood.
- Gomes, M. and J. da Silva (1999). An experimental evaluation of the GRASP metaheuristic applied to the uncapacitated location problem. Technical Report 004/99, Department of Statistics and Computation, State University of Ceará, Fortaleza, Ceará, Brazil.
 Two GRASP heuristics, one using the ADD heuristic and the other using the DROP heuristic, are proposed for the uncapacitated location problem. Computational experiments with instances from Beasley's OR-Library show that GRASP-DROP dominates GRASP-ADD, while both GRASP heuristics dominate ADD and DROP.
- Holmqvist, K., A. Migdalas, and P. Pardalos (1997). Greedy randomized adaptive search for a location problem with economies of scale. In I. B. et al. (Ed.), *Developments in Global Optimization*, pp. 301–313. Kluwer Academic Publishers.
 A GRASP is proposed for finding approximate solutions to a facility location problem with concave costs. The greedy function of the construction phase favors the facilities

that give lower cost for a customer, regarding the effect that already connected customers have on the solution. The neighborhood function is defined as changing facility connection for one customer. Instead of a time consuming computation of the objective function value for each neighborhood solution, the difference in cost for changing supplier is examined.

Klincewicz, J. (1992). Avoiding local optima in the p -hub location problem using tabu search and GRASP. *Annals of Operations Research* **40**, 283–302.

Two heuristics are proposed, based on tabu search and GRASP, for the p -hub location problem. The objective is to overcome the difficulty that local search algorithms encounter. See also page 39.

Klincewicz, J. (2002). Enumeration and search procedures for a hub location problem with economies of scale. *Annals of Operations Research* **110**, 107–122.

An optimal enumeration scheme, as well as other heuristics based on tabu search and GRASP are proposed for locating hubs in a communications or transportation network. See also page 39.

Kumaran, K., A. Srinivasan, Q. Wang, S. Lanning, and K. Ramakrishnan (2001). Efficient algorithms for location and sizing problems in network design. In *Global Telecommunications Conference, 2001 (GLOBECOM '01)*, Volume 4, pp. 2586–2590. IEEE.

Algorithms based on linear programming and a slightly modified GRASP are developed. The construction phase is performed at random. Given an initial solution, the local search procedure exhaustively evaluate objective function value for all possible single location changes. See also page 40.

Resende, M. and R. Werneck (2003). A hybrid multistart heuristic for the uncapacitated facility location problem. Technical report, Internet and Network Systems Research Center, AT&T Labs Research, Florham Park, NJ.

A multistart heuristic is proposed for the uncapacitated facility location problem, based on an idea proposed for the p -median problem in Resende and Werneck (2004). The algorithm consists in two phases. The first phase is a multistart procedure that builds a randomized solution from which to apply a local search routine and a path-relinking. The second is a post-optimization phase realized by applying path-relinking over the whole elite set. See also page 9.

Resende, M. and R. Werneck (2004). A hybrid heuristic for the p -median problem. *Journal of Heuristics* **10**, 59–88.

A GRASP with path-relinking as intensification and post-optimization phase is proposed for the p -median problem. See also page 9.

Serra, D. and R. Colomé (2001). Consumer choice in competitive location models: Formulations and heuristics. *Papers in Regional Science* **80**, 439–464.

The importance of customer behavior with respect to distance or transportation costs in the optimality of locations obtained by traditional state-of-the-art competitive location models is addressed. Four models to represent the problem are proposed. A hybrid metaheuristic is proposed.

Silva, F. and D. Serra (2002). Locating emergency services with priority rules: The priority queuing covering location problem. Technical Report 1, Research group in management logistics, Department of Economics and Business, Universitat Pompeu Fabra, Barcelona, Spain.

The greedy criterion used by the proposed GRASP is based on distances. For each server, one at a time, the local search iteratively de-allocates all demands allocated to it and moves them to all possible unused candidates.

Urban, T. (1998). Solution procedures for the dynamic facility layout problem. *Annals of Operations Research*, 323–342.

The concept of incomplete dynamic programming is applied to the dynamic facility layout problem and a lower bound for the general problem is developed. A GRASP and an initialized multi-greedy algorithm are described to provide a solution methodology for large problems. The GRASP is the algorithm proposed by Li, Pardalos, and Resende (1994) for dense quadratic assignment problems.

Urban, T., W.-C. Chiang, and R. Russel (2000). The integrated machine allocation and layout problem. *International Journal of Production Research* **38**, 2911–2930.

GRASP is used to solve quadratic assignment sub-problems in a model that aggregates quadratic assignment problems with several network flow problems with side constraints. See also page 22.

10. COVERING, CLUSTERING, PACKING, AND PARTITIONING

Covering, clustering, packing, and partitioning are problem areas where GRASP has been recently successfully applied. The papers below illustrate this.

Ahmadi, S. and I. Osman (2003). Greedy random adaptive memory programming search for the capacitated clustering problem. *European Journal of Operational Research*. To appear.

A GRAMPS framework, which is a hybrid of GRASP and AMP (Adaptive Memory Programming), is proposed for the capacitated clustering problem. See also page 6.

Areibi, S. (1999). GRASP: An effective constructive technique for VLSI circuit partitioning. In *Proc. IEEE Canadian Conference on Electrical & Computer Engineering (CCECE'99)*, Volume 1, pp. 462–467.

At each iteration of the proposed randomized approach, the gains associated with moving modules to the current block being filled are examined and a restricted candidate list is built using the modules with the highest gains. See also page 23 and page 45.

Areibi, S. and A. Vannelli (2000). Efficient hybrid search techniques for circuit partitioning. In *IEEE 4th World Multiconference on Circuits, Systems, Communications & Computers*.

Some pure metaheuristics and as well as heuristic hybridizations are proposed for circuit partitioning. A GRASP and a genetic algorithm are used for generating good initial partitions. See also page 7, page 23, and page 45.

Argüello, M., T. Feo, and O. Goldschmidt (1996). Randomized methods for the number partitioning problem. *Computers & Operations Research* **23**(2), 103–111.

Randomized methodologies for solving the number partitioning problem are proposed. The greedy criterion consists in considering only large elements for differencing. Specific selection of the elements to be differenced is made at random. Differences are placed back into the list of remaining elements, and the process of selecting the next element is repeated. The proposed methods are greedy, randomized, and adaptive construction heuristics, but local search is omitted.

Cano, J., R. Cabrera, and J. Vega (2002). Procedimientos constructivos adaptivos (GRASP) para el problema del empaquetado bidimensional. *Revista Iberoamericana de Inteligencia Artificial* **15**, 26–33.

To solve the bidimensional packing problem, several constructive adaptive heuristics are proposed. Some of them only have a GRASP construction phase, while others apply also a local search phase. Computational results show that in many cases the proposed heuristics obtain the optimal solution. In Spanish.

Cano, J., O. Cordon, F. Herrera, and L. Sánchez (2002a). A greedy randomized adaptive search procedure applied to the clustering problem as an initialization process using K-means as a local search procedure. *Journal of Intelligent and Fuzzy Systems* **12**, 235–242.

A GRASP is proposed for cluster analysis using a probabilistic greedy Kaufman initialization in the construction phase and K-Means as local search procedure.

Cano, J., O. Cerdón, F. Herrera, and L. Sánchez (2002b). A GRASP algorithm for clustering. In F. J. Garijo, J. Santos, and M. Toro (Eds.), *Advances in Artificial Intelligence - IBERAMIA 2002, 8th Ibero-American Conference on AI, Seville, Spain, November 12-15, 2002, Proceedings*, Volume 2527 of *Lecture Notes in Computer Science*, pp. 214–223. Springer.

A GRASP for cluster analysis is described. See also page 7 and page 24.

Catalano, M. F. and F. Malucelli (2001). Parallel randomized heuristics for the set covering problem. *International J. of Computer Research* **10**(4), 113–132.

A general scheme to design heuristics for the set covering problem is proposed. A first group of procedures randomize the choice of the next element to be added at the solution under construction in a way similar to ant system, while a second set of procedures introduces a random perturbation of the costs of the problem instance. The second set includes also a GRASP.

Chardaire, P., G. McKeown, and J. Maki (2001). Application of GRASP to the multiconstraint knapsack problem. In E. B. et al. (Ed.), *EvoWorkshop 2001*, pp. 30–39. Springer-Verlag Berlin Heidelberg. Several implementations of GRASP for the multiconstraint knapsack problem are presented. In all implementations, the greedy functions are based on the profit per weight unit associated with each element. 1-opt and 2-opt search strategies are used in the local search phase.

Delorme, X., X. Gandibleux, and J. Rodriguez (2001). GRASP for set packing problems. In *Proceedings of the Operational Research Peripatetic Post-Graduate Programme (ORP3)*.

Two GRASP implementations for the set packing problem are proposed. The first GRASP is inspired by a GRASP for the set covering problem that appeared in the literature. The neighborhood structure adopted is a k - p exchange, which consists in fixing to 0 the value of k variables and to 1 the value of the remaining p variables. The 0-1, 1-1, 2-1, and 1-2 exchange neighborhoods are investigated. The second GRASP is inspired by a GRASP for the node packing problem that appeared in the literature and uses a 1-2 exchange neighborhood.

Delorme, X., X. Gandibleux, and J. Rodriguez (2004). GRASP for set packing problems. *European J. of Operational Research* **153**(3), 564–580.

GRASP is applied to solve the set packing problem. Several construction phases are studied and improvements based on advanced strategies are evaluated. These include reactive GRASP, path relinking, and a procedure involving the diversification of the selection (using a learning process).

Feo, T. and M. Resende (1989). A probabilistic heuristic for a computationally difficult set covering problem. *Operations Research Letters* **8**, 67–71.

GRASP is proposed for a class of difficult set covering problems that arise in computing the 1-width of the incidence matrix of Steiner triple systems. A value based restricted candidate list is used in the construction phase. The local search is based on the elimination of redundant elements in the cover. See also page 2.

Hammer, P. and D. Rader, Jr. (2001). Maximally disjoint solutions of the set covering problem. *Journal of Heuristics* **7**, 131–144.

The problem of finding two solutions of a set covering problem that have a minimum number of common variables is addressed. It is proved that this problem is NP-complete and three heuristics are proposed for solving it. Two of these algorithms find the solutions sequentially. One of them is a GRASP.

Klincewicz, J. and A. Rajan (1994). Using GRASP to solve the component grouping problem. *Naval Research Logistics* **41**, 893–912.

Two new heuristics are proposed for solving a particular set partitioning problem that arises in robotics assembly, as well as in a number of other manufacturing and material logistics application areas. The heuristics are GRASPs involving two alternate procedures for determining starting points: component-based and code-based. See also page 36.

Laguna, M., T. Feo, and H. Elrod (1994). A greedy randomized adaptive search procedure for the two-partition problem. *Operations Research* **42**, 677–687.

A GRASP for the network 2-partition problem is proposed. The greedy function of the construction phase minimizes the augmented weight of the partition. For the local improvement phase, four alternative procedures are considered: best swap, first swap, slight swap, and slightest swap. The best strategies are slight and slightest swaps. Slight swap selects a near-minimum gain exchange at each iteration, while slightest swap chooses the absolute minimum gain.

Pacheco, J. and O. Valencia (2003). Design of hybrids for the minimum sum-of-squares clustering problem. *Computational Statistics and Data Analysis* **43**, 235–248.

Several heuristics are proposed for the non-hierarchical clustering problem under the criterion of minimum sum-of-squares clustering. These heuristics incorporate genetic operators, local search, and tabu search. They are compared with other heuristic approaches, including a GRASP, on a set of test problems. See also page 8.

Resende, M. (1998). Computing approximate solutions of the maximum covering problem using GRASP. *Journal of Heuristics* **4**, 161–171.

A GRASP for the maximum covering problem is described. For details about the GRASP implementation, see page 41.

11. ROUTING

Routing problems arise in transportation, telecommunications, and waste management. Such applications are contained in the papers below.

Arakaki, R. (1998). O problema de roteamento de veículos e algumas metaheurísticas. Master's thesis, Instituto Nacional de Pesquisas Espaciais, Brazil.

Several metaheuristics are proposed for the vehicle routing problem, including a GRASP. In the GRASP construction phase, a distance function is used as the greedy function, while the local search tries to find a better allocation for a customer at a time. In Portuguese.

Argüello, M., J. Bard, and G. Yu (1997). A GRASP for aircraft routing in response to groundings and delays. *Journal of Combinatorial Optimization* **1**, 211–228.

A GRASP is presented to reconstruct aircraft routings in response to groundings and delays experienced over the course of the day. The objective is to minimize the cost of reassigning aircraft to flights taking into account available resources and other system constraints. See also page 36.

Baker, B. and C. Carreto (2003). A visual interactive approach to vehicle routing. *Computers and Operations Research* **30**, 321–337.

A graphical-user-interface (with a Microsoft Windows interface) and a GRASP based heuristic for the basic vehicle routing problem are described. The proposed visual interactive system, CRUISE, provides high flexibility. Although the best known results on VRP benchmarks are obtained by tabu search and simulated annealing algorithms, none

of them allows the user any control for combining their insights and knowledge. See also page 36.

- Bard, J., L. Huang, P. Jaillet, and M. Dror (1998). A decomposition approach to the inventory routing problem with satellite facilities. *Transportation Science* **32**, 189–203.

A methodology is presented that decomposes the inventory routing problem with satellite facilities over the planning horizon, and then solves daily rather than multi-day vehicle routing problems. Three heuristics are proposed for solving the vehicle routing problem with satellite facilities: randomized Clarke-Wright, GRASP, and modified sweep. See also page 37.

- Bard, J., G. Kontoravdis, and G. Yu (2002). A branch-and-cut procedure for the vehicle routing problem with time windows. *Transportation Science* **36**, 250–269.

A GRASP is proposed to obtain feasible solutions and/or upper bounds used in a branch-and-cut algorithm for the vehicle routing problem with time windows. See also page 37.

- Carreto, C. and B. Baker (2002). A GRASP interactive approach to the vehicle routing problem with backhauls. In C. Ribeiro and P. Hansen (Eds.), *Essays and surveys in metaheuristics*, pp. 185–200. Kluwer Academic Publishers.

A GRASP for the vehicle routing problem with backhauls is proposed. The construction phase is implemented in a clustering heuristic that constructs the routes by clustering the remaining customers according to the vehicles defined by seeds while applying the 3-opt heuristic to reduce the total distance traveled by each vehicle. The greedy function takes into account routes with smallest insertion cost and costumers with biggest difference between the smallest and the second smallest insertion costs and smallest number of routes they can traverse. As the local search phase, 3-opt is used. See also page 8.

- Chaovalitwongse, W., D. Kim, and P. Pardalos (2003). GRASP with a new local search scheme for vehicle routing problems with time windows. *J. of Combinatorial Optimization* **7**, 179–207.

A GRASP is proposed for minimizing the number of needed vehicles and the travel distances in the vehicle routing problem with time windows. The search method proposed uses a combination of random and greedy functions.

- Corberán, A., R. Martí, and J. Sanchís (2002). A GRASP heuristic for the mixed Chinese postman problem. *European Journal of Operational Research* **142**, 70–80.

The construction phase of the GRASP proposed to solve the mixed Chinese postman problem uses a greedy function based on the definition of the degree of a node in terms of both incident oriented arcs and incident undirected edges. Each iteration of the local search procedure selects a pair of vertices $u, v \in V$ that are candidates for the move if they are joined by a path of duplications. See also page 15.

- Hjorring, C. (1995). *The vehicle routing problem and local search metaheuristics*. Ph. D. thesis, University of Auckland, Auckland, New Zealand.

Three metaheuristics for effectively searching through the space of cyclic orders are developed. They are based on GRASP, tabu search, and genetic algorithms. For tabu search, different schemes are investigated to control the tabu list length, including a reactive tabu search method. To obtain good solutions when using the genetic algorithm, specialized crossovers are developed, and a local search component is added. GRASP is used to construct an initial good solution.

- Kontoravdis, G. and J. Bard (1995). A GRASP for the vehicle routing problem with time windows. *ORSA Journal on Computing* **7**, 10–23.

A GRASP is proposed for minimizing the fleet size of temporarily constrained vehicle routing problems with two types of service. The greedy function of the construction phase takes into account both the overall minimum insertion cost and the penalty cost.

Local search is applied to the best solution found every five iterations of the first phase, rather than to each feasible solution.

Marinakis, Y. and A. Migdalas (2003). Expanding neighborhood GRASP for the Traveling Salesman Problem. Technical report, Technical University of Crete, Chania 73100, Greece.

A GRASP that includes tour improvement methods is proposed for the traveling salesman problem. See also page 8 and page 16.

Resende, L. and M. Resende (1999). A GRASP for frame relay PVC routing. In *Proc. of the Third Metaheuristics International Conference*, pp. 397–402.

A GRASP is described for routing permanent virtual circuits (PVC) for frame relay in telecommunications systems. The objective is to minimize PVC delays while balancing trunk loads. The greedy choice selects from the set of not yet routed PVCs the one that minimizes the delay while balancing the trunk loads. See also page 41.

Resende, M. and C. Ribeiro (2003). GRASP with path-relinking for private virtual circuit routing. *Networks* **41**, 104–114.

A frame relay service offers virtual private networks to customers by provisioning a set of long-term private virtual circuits (PVCs) between customer endpoints on a large backbone network. During the provisioning of a PVC, routing decisions are made without any knowledge of future requests. Over time, these decisions can cause inefficiencies in the network and occasional offline rerouting of the PVCs is needed. The offline PVC routing problem is formulated as an integer multicommodity flow problem with additional constraints and with an objective function that minimizes delays and network overload. Variants of a GRASP with path-relinking heuristic are proposed for this problem. See also page 9 and page 41.

12. SEQUENCING AND SCHEDULING

GRASP has been applied to numerous sequencing and scheduling problems. The papers in this section illustrate this.

Aiex, R. (2002). *Uma investigação experimental da distribuição de probabilidade de tempo de solução em heurísticas GRASP e sua aplicação na análise de implementações paralelas*. Ph. D. thesis, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil.

A new methodology is described for the analysis of GRASP. Hybrid strategies with path-relinking are also proposed. These are studied on the 3-index assignment problem as well as the job shop scheduling problem. See also page 11 and page 18.

Aiex, R., S. Binato, and M. Resende (2003). Parallel GRASP with path-relinking for job shop scheduling. *Parallel Computing* **29**, 393–430.

A parallel GRASP with path-relinking as an intensification strategy for the job shop problem is described based on some ideas proposed in the GRASP of Binato et al. (2002). See also page 6 and page 11.

Akturk, M. and K. Kiliç (1999). Generating short-term observations for space mission projects. *J. of Intelligent Manufacturing* **10**, 387–404.

Generating short-term observations for space mission projects is basically a scheduling problem. It consists in generating short-term observation schedules of the Hubble Space Telescope (HST) such that the scientific return is maximized. A new dispatching rule and a set of local search based algorithms (including a GRASP) are proposed.

Akturk, M. and D. Ozdemir (2001). A new dominance rule to minimize total weighted tardiness with unequal release dates. *European Journal of Operational Research* **135**, 394–412.

A new algorithm that provides sufficient condition for local optimality and that can be embedded into several heuristic frameworks (including GRASP) is proposed.

- Atkinson, J. (1998). A greedy randomised search heuristic for time-constrained vehicle scheduling and the incorporation of a learning strategy. *Journal of the Operational Research Society* **49**, 700–708.

Two forms of adaptive search called *local* and *global* adaptation are identified. In both search techniques, the greedy function takes into account a quantity that measures heuristically the quality of the partial solution. While in local adaptation the decisions made within a particular run influence only the subsequent performance of the heuristic, global adaptation involves making decisions that affect the performance of the heuristic in subsequent runs. See also page 36.

- Bard, J. and T. Feo (1989). Operations sequencing in discrete parts manufacturing. *Management Science* **35**, 249–255.

A method for efficiently sequencing cutting operations associated with the manufacture of discrete parts is proposed. The problem is modeled as an integer program. This is relaxed via Lagrangian relaxation into a min-cut problem on a bipartite network. To obtain lower bounds, a max-flow algorithm is applied and the corresponding solution is input to a GRASP. See also page 35.

- Bard, J., T. Feo, and S. Holland (1996). A GRASP for scheduling printed wiring board assembly. *I.I.E. Transactions* **28**, 155–165.

The assembly of printed wiring boards (PWBs) typically involves the coordination of thousands of components and hundreds of part numbers in a job shop environment with more than 50 different processes and workstations. A GRASP is proposed for solving the daily scheduling problem that arises in such environment. See also page 35.

- Binato, S., W. Hery, D. Loewenstern, and M. Resende (2002). A greedy randomized adaptive search procedure for job shop scheduling. In C. Ribeiro and P. Hansen (Eds.), *Essays and surveys in metaheuristics*, pp. 58–79. Kluwer Academic Publishers.

A GRASP is designed, incorporating an intensification strategy and a POP (Proximate Optimality Principle) in the construction phase. The greedy criterion is to minimize the makespan resulting from the addition of an operation to the schedule under construction, while the local search procedure uses a 2-exchange neighborhood.

- Casey, S. and J. Thompson (2003). GRASPing the examination scheduling problem. In E. Burke and P. D. Causmaecker (Eds.), *PATAT 2002*, Volume 2740 of *Lecture Notes in Computer Science*, pp. 232–244. Springer.

Reactive GRASP is applied to the examination scheduling problem. A comparison with other algorithms for this problem shows that GRASP is a powerful solution method.

- Christofoletti, L. (2002). Métodos de reinício aplicados ao seqüenciamento em uma máquina com tempos de preparação e data de entrega. Master's thesis, Departamento de Engenharia de Sistemas, Universidade Estadual de Campinas, Brazil.

This thesis considers applications of GRASP and its variants to the problem of minimizing the total tardiness of jobs on a single machine with sequence dependent setup times. An innovative aspect of this thesis is the introduction in the multistart framework with some memory strategies for storing a population of high quality solutions. In Portuguese.

- Christofoletti, L. and V. Armentano (2001). Estratégias de reinício de busca local baseadas em memória para programação de tarefas em uma máquina. In *Proceedings of the XXXIII Brazilian Symposium on Operations Research*, pp. 1381–1389.

It is shown that incorporating adaptive memory in multistart random methods improves their performance. To validate their thesis, the paper addresses the problem of minimizing total job tardiness on a single machine with sequence dependent setup times. A

GRASP is implemented, combined with some basic principles of memory utilization during the construction phase. In Portuguese.

De, P., J. Ghosj, and C. Wells (1994). Solving a generalized model for CON due date assignment and sequencing. *International Journal of Production Economics* **34**, 179–185.

A generalized model for assigning a constant flow allowance (CON) due date to a set of jobs and sequencing them on a single machine is considered. The problem is viewed as a 0-1 quadratic problem and a GRASP is proposed to solve the quadratic problem. The randomization strategy used is inspired by a gradient-based variable forcing methodology proposed by Pardalos and Rodgers (1990) for a branch & bound algorithm. The local search procedure is based on a definition of neighborhood in which two solutions are neighbors if they differ in the value of exactly one variable.

Feo, T. and J. Bard (1989). Flight scheduling and maintenance base planning. *Management Science* **35**, 1415–1432.

A model is presented that can be used by planners to both locate maintenance stations and develop flight schedules that better meet the cyclical demand for maintenance. See also page 37.

Feo, T., J. Bard, and S. Holland (1995). Facility-wide planning and scheduling of printed wiring board assembly. *Operations Research* **43**, 219–230.

A decision support system known as INSITES is described. INSITES was designed to assist Texas Instruments in the day-to-day assembly operations of their printed wiring board (PWB) facilities. A GRASP is used to solve the underlying multiple machine scheduling problem. See page 36 for details of the GRASP implementation.

Feo, T., K. Sarathy, and J. McGahan (1996). A GRASP for single machine scheduling with sequence dependent setup costs and linear delay penalties. *Computers & Operations Research* **23**, 881–895.

A GRASP for single machine scheduling with sequence dependent setup costs and linear delay penalties is presented. The greedy function of the GRASP construction phase proposed is made up of two components: the switch over cost and the opportunity cost associated with not inserting a specific job in the next position and instead, inserting it after half of the unscheduled jobs have been scheduled. This greedy function tends to lead to a balance between the natural order and nearest neighbor approaches. The local search uses 2-exchange, insertion exchange, and a combination of the two.

Feo, T., K. Venkatraman, and J. Bard (1991). A GRASP for a difficult single machine scheduling problem. *Computers & Operations Research* **18**, 635–643.

GRASP is applied to an unusually difficult scheduling problem with flow time and earliness penalties. Two greedy functions are developed and tested. The first is the difference between the flow time and earliness penalties, normalized by the processing time. The second function evaluates the cost of scheduling a job next by estimating the cost of the remaining schedule. The local search uses 2-exchange and insertion exchange.

García, J., S. Lozano, K. Smith, and F. Guerrero (2001). A comparison of GRASP and an exact method for solving a production and delivery scheduling problem. In *First International Workshop on Hybrid Intelligent Systems (HIS'01)*, Adelaide, Australia.

An exact approach and a GRASP are proposed to solve a production and delivery scheduling problem. The greedy criterion takes into account order weights, while the local search procedure uses an exchange neighborhood. See also page 37.

Laguna, M. and J. González-Velarde (1991). A search heuristic for just-in-time scheduling in parallel machines. *Journal of Intelligent Manufacturing* **2**, 253–260.

A hybrid GRASP/tabu search metaheuristic is proposed for the weighted earliness penalty problem with deadlines in identical parallel machines.

Lourenço, H., J. Paixão, and R. Portugal (2001). Multiobjective metaheuristics for the bus-driver scheduling problem. *Transportation Sciences* **35**, 331–343.

Several metaheuristics are presented to solve real driver scheduling problems in public transportation bus companies. They include a GRASP. See also page 38.

Ríos-Mercado, R. and J. Bard (1998). Heuristics for the flow line problem with setup costs. *European Journal of Operational Research*, 76–98.

Two new heuristics are presented for the flowshop scheduling problem with sequence-dependent setup times and makespan minimization objective, one of which is a GRASP.

Rivera, L. (1998). Evaluation of parallel implementations of heuristics for the course scheduling problem. Master's thesis, Instituto Tecnológico y de Estudios Superiores de Monterrey, Monterrey, Mexico.

Several parallel implementations of heuristics are proposed for the course scheduling problem, including a GRASP. In Spanish. See also page 13.

Rojanasoonthon, S. and J. Bard (2004). A GRASP for parallel machine scheduling with time windows. *INFORMS Journal on Computing*. To appear.

A GRASP is designed for a parallel machine scheduling problem with time windows. This extends the GRASP proposed in Rojanasoonthon, Bard, and Reddy (2003) to solve parallel machine scheduling problems in the presence of time windows.

Rojanasoonthon, S., J. Bard, and S. Reddy (2003). Algorithms for parallel machine scheduling: A case study of the tracking and data relay satellite system. *Journal of the Operational Research Society* **54**, 806–821.

A GRASP is designed for a parallel machine scheduling problem. The greedy criterion in the construction phase is based on maximizing a flexibility function that measures how much slack a schedule has after the insertion is made.

Souza, M., N. Maculan, and L. Ochi (2003). A GRASP-tabu search algorithm for school timetabling problems. In M. Resende and J. de Sousa (Eds.), *Metaheuristics: Computer decision-making*, pp. 659–672. Kluwer Academic Publishers.

A hybrid approach is proposed for school timetabling problems. It uses a greedy randomized construction phase for obtaining a feasible solution to be possibly improved by applying a tabu search. The greedy choice first takes into account the number of available teachers and then the activity degree of each teacher. A timetable is represented as a $m \times q$ matrix Q of integer values, such that each row i represents the weekly schedule of teacher i and q_{ik} represents the activity of teacher i in period k . A neighbor of a timetable Q is a timetable Q' , obtained from Q simply by changing two different and nonnegative values of a give row of Q . See also page 10.

Xu, J. and S. Chiu (1996). Solving a real-world field technician scheduling problem. In *Proceedings of the International Conference on Management Science and the Economic Development of China*, pp. 240–248.

The objective of the field technician scheduling problem is to assign a set of jobs at different locations with time windows to technicians with different job skills. The greedy choice of the proposed GRASP is to select jobs with the highest unit weight. See also page 42.

Xu, J. and S. Chiu (2001). Effective heuristic procedure for a field technician scheduling problem. *Journal of Heuristics* **7**, 495–509.

Several heuristics, including a GRASP, are designed and tested for solving the field technician scheduling problem. See also page 42.

13. LOGIC

GRASP has been applied to problems in logic, including SAT, MAX-SAT, and logical clause inference, as shown by the following papers.

de Campos, L., J. Fernández-Luna, and J. Puerta (2002). Local search methods for learning Bayesian networks using a modified neighborhood in the space of DAGs. In F. J. Garijo, J. Riquelme, and M. Toro (Eds.), *Advances in Artificial Intelligence - IBERAMIA 2002, 8th Ibero-American Conference on AI, Seville, Spain, November 12-15, 2002, Proceedings*, Volume 2527 of *Lecture Notes in Computer Science*, pp. 182–192. Springer.

GRASP is applied to learning Bayesian networks from data. The GRASP construction phase is a randomization of the greedy algorithm of Butine (1991) and the local search is a hill-climbing algorithm in the space of directed acyclic graphs (DAGs). The GRASP obtains excellent results in the computational experiments described.

Deshpande, A. and E. Triantaphyllou (1998). A greedy randomized adaptive search procedure (GRASP) for inferring logical clauses from examples in polynomial time and some extensions. *Mathematical and Computer Modelling* **27**, 75–99.

Two heuristics (one of which is a GRASP) are presented for inferring a small size Boolean function from complete and incomplete examples in polynomial time. Each example can be positive or negative depending on whether it must be accepted or rejected, respectively, by the target function. Both of the proposed heuristics are randomized in the sense that instead of choosing the best candidate element, a candidate list is built whose elements are assigned with evaluative function values close to the highest one.

Pardalos, P., L. Pitsoulis, and M. Resende (1996). A parallel GRASP for MAX-SAT problems. *Lecture Notes in Computer Science* **1184**, 575–585.

A parallel GRASP for weighted maximum satisfiability (MAX-SAT) problem is proposed. The GRASP is based on the serial GRASP presented by Resende, Pitsoulis, and Pardalos (1997). See also page 13.

Resende, M. and T. Feo (1996). A GRASP for satisfiability. In D. Johnson and M. Trick (Eds.), *Cliques, Coloring, and Satisfiability: The Second DIMACS Implementation Challenge*, Volume 26 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pp. 499–520. American Mathematical Society.

A GRASP is described for the satisfiability problem. It can be also directly applied to both the weighted and unweighted versions of the maximum satisfiability problem. The adaptive greedy function is a hybrid combination of two functions. One function seeks to maximize the number of yet-unsatisfied clauses that become satisfied after the assignment of each construction iteration, while the other maximizes the number of yet-unassigned literals in yet-unsatisfied clauses that become satisfied if opposite assignments were to be made. The local search flips the assignment of each variable, one at a time, checking if the new truth assignment increases the number of satisfied clauses.

Resende, M., L. Pitsoulis, and P. Pardalos (1997). Approximate solution of weighted MAX-SAT problems using GRASP. In J. Gu and P. Pardalos (Eds.), *Satisfiability problems*, Volume 35 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pp. 393–405. American Mathematical Society.

A GRASP is proposed for finding approximate solutions of weighted MAX-SAT problems. The greedy adaptive function is to maximize the total weight of yet-unsatisfied clauses that become satisfied after the assignment of each construction phase iteration. The local search uses the 1-flip neighborhood of a vector x , defined as the set of all binary vectors that differ from x in exactly one literal.

Resende, M., L. Pitsoulis, and P. Pardalos (2000). Fortran subroutines for computing approximate solutions of MAX-SAT problems using GRASP. *Discrete Applied Mathematics* **100**, 95–113.

A set of Fortran subroutines for computing approximate solutions of MAX-SAT problems is described. The algorithm implemented was proposed by Resende, Pitsoulis, and Pardalos (1997). Two versions of the subroutines are distributed. One version uses a

neighborhood data structure in order to speed up the local search phase, while the second version, since it does not make use of this data structure, is more memory efficient but less time efficient. Computational results improve upon those in Resende, Pitsoulis, and Pardalos (1997) using an RCL parameter α randomly chosen each GRASP iteration from the interval $[0, 1]$. See also page 11.

Yilmaz, E., E. Triantaphyllou, J. Chen, and T. Liao (2003). A heuristic for mining association rules in polynomial time. *Mathematical and Computer Modelling* **37**, 219–233.

The problem consisting in mining association rules in a database needs to be efficiently solved, especially nowadays with modern very large databases. The authors propose a heuristic algorithm that incorporates the randomized idea of the GRASP construction phase.

14. MANUFACTURING

GRASP has been used to address several applications in manufacturing. The following papers are examples of this.

Bard, J. and T. Feo (1989). Operations sequencing in discrete parts manufacturing. *Management Science* **35**, 249–255.

An integer program is relaxed via Lagrangian relaxation into a min-cut problem on a bipartite network. To obtain lower bounds, a max-flow algorithm is applied and the corresponding solution is input to a GRASP. See also page 31.

Bard, J. and T. Feo (1991). An algorithm for the manufacturing equipment selection problem. *IIE Transactions* **23**, 83–92.

The objective is to determine how many of each machine type to purchase and what fraction of the time each piece of equipment will be configured for a particular type of operation. The problem is converted into a MILP and a depth-first branch & bound algorithm is used, employing the greedy randomized set covering heuristic of Feo and Resende (1989), to implicitly search for optimality.

Bard, J., T. Feo, and S. Holland (1996). A GRASP for scheduling printed wiring board assembly. *I.I.E. Transactions* **28**, 155–165.

A GRASP is proposed for solving the daily scheduling problem that arises in a job shop environment with more than 50 different processes and workstation. See also page 31.

Bautista, J., R. Suárez, M. Mateo, and R. Companys (2000). Local search heuristics for the assembly line balancing problem with incompatibilities between tasks. In *Proceedings of the IEEE ICRA-00*, Volume 3, pp. 2404–2409.

The assembly line balancing problem with incompatibilities between tasks consists in minimizing the total number of needed workstations and minimizing the cycle time for the minimum number of workstations. A GRASP and a genetic algorithm are proposed for solving the problem. The greedy choice favors tasks with the best index value, while the local search phase simply changes the order of elements in the sequence solution.

Feo, T. and J. Bard (1989). The cutting path and tool selection problem in computer-aided process planning. *Journal of Manufacturing Systems* **8**, 17–26.

A method for minimizing the sum of tool setup and volume removal times associated with metal cutting operations on a flexible machine is given. The problem is modeled as an integer program, then relaxed into a min-cut problem on a simple network. After obtaining a tentative solution, a GRASP is used to identify good feasible points corresponding to alternative process plans. These are seen to speed convergence during branch & bound.

Feo, T., J. Bard, and S. Holland (1995). Facility-wide planning and scheduling of printed wiring board assembly. *Operations Research* **43**, 219–230.

A GRASP is used to solve a multiple machine scheduling problem. The schedule produced at each GRASP iteration is evaluated based on one of five different optimization criteria. The choice of the criterion to be followed is made by the user to rank order the schedules provided by multiple GRASP iterations. See page 32.

Klincewicz, J. and A. Rajan (1994). Using GRASP to solve the component grouping problem. *Naval Research Logistics* **41**, 893–912.

Two new GRASPs are proposed that involve two alternate procedures for determining starting points: component-based and code-based. See also page 28.

Yen, J., M. Carlsson, M. Chang, J. Garcia, and H. Nguyen (2000). Constraint solving for inkjet print mask design. *Journal of Imaging Science and Technology* **44**, 391–397.

Print masks are used to determine which nozzles on an inkjet printer cartridge are to spit an ink droplet at each particular instant in a multiple-pass print mode. A GRASP is proposed for for automatic generation of print masks and has been used to design the print masks for Hewlett Packard’s wide format printers (DeskJet 2500C and 2500CM).

15. TRANSPORTATION

GRASP has been used to find approximate solutions of problems in air, rail, and intermodal transportation. The following papers illustrate these applications.

Argüello, M., J. Bard, and G. Yu (1997). A GRASP for aircraft routing in response to groundings and delays. *Journal of Combinatorial Optimization* **1**, 211–228.

A neighborhood search technique is proposed that takes as input an initial feasible solution, so that the construction phase is omitted. Two types of partial route exchange operations are described. The first exchanges flight sequences with identical endpoints and in the second sequence of flights being exchanged must have the same origination airport, but the termination airports are swapped. See also page 28.

Atkinson, J. (1998). A greedy randomised search heuristic for time-constrained vehicle scheduling and the incorporation of a learning strategy. *Journal of the Operational Research Society* **49**, 700–708.

In both adaptive search techniques proposed and called *local* and *global* adaptations, respectively, the greedy function takes into account a quantity that measures heuristically the quality of the partial solution. While in local adaptation the decisions made within a particular run influence only the subsequent performance of the heuristic, global adaptation involves making decisions that affect the performance of the heuristic in subsequent runs. See also page 31.

Baker, B. and C. Carreto (2003). A visual interactive approach to vehicle routing. *Computers and Operations Research* **30**, 321–337.

See page 28.

Bard, J. (1997). An analysis of a rail car unloading area for a consumer products manufacturer. *Journal of the Operational Research Society* **48**, 873–883.

Discussion of design and analysis of the railcar unloading area of Procter & Gamble’s principal laundry detergent plant. To solve the problem, four alternatives are proposed and evaluated with the help of a GRASP. See also page 19.

Bard, J., L. Huang, P. Jaillet, and M. Dror (1998). A decomposition approach to the inventory routing problem with satellite facilities. *Transportation Science* **32**, 189–203.

Three heuristics are proposed for solving the vehicle routing problem with satellite facilities: randomized Clarke-Wright, GRASP, and modified sweep. The GRASP proposed is a modified version of the GRASP of Kontoravdis and Bard (1995). See also page 29.

Bard, J., G. Kontoravdis, and G. Yu (2002). A branch-and-cut procedure for the vehicle routing problem with time windows. *Transportation Science* **36**, 250–269.

A GRASP is proposed to obtain feasible solutions and/or upper bounds used in a branch-and-cut algorithm for the vehicle routing problem with time windows. See also page 29.

Campbell, A. and M. Savelsbergh (2002). Decision support for consumer direct grocery initiatives. Technical report, Department of Management Sciences, Tippie College of Business, University of Iowa.

Business-to-Consumer e-commerce has led to the proposal of new consumer direct service models and activities, such as grocery delivery services. The seller has to decide which request to accept and for each accepted request he has to establish the time slot when the delivery is going to be done. Insertion based heuristics are proposed. To improve the chances that a delivery request can be taken, randomization is used as in the GRASP proposed by Kontoravdis and Bard (1995).

Campbell, A. and M. Savelsbergh (2003). Incentive schemes for consumer direct delivery. Technical report, Department of Management Sciences, Tippie College of Business, University of Iowa.

A different aspect of a problem arising in Business-to-Consumer e-commerce is addressed, i.e the promise of a delivery window. Several approaches are proposed, including a GRASP. The greedy criterion is based on the costs of inserting a delivery into a feasible schedule.

Delorme, X., J. Rodriguez, and X. Gandibleux (2001). Heuristics for railway infrastructure saturation. In C. Zaroliagis (Ed.), *Electronic Notes in Theoretical Computer Science*, Volume 50. Elsevier.

To evaluate railway infrastructure capacity, two heuristics approaches are proposed, including a GRASP. The greedy function is defined on the number of mathematical model constraints concerned by decision variables, while local search procedure uses a k - p exchange neighborhood.

Feo, T. and J. Bard (1989). Flight scheduling and maintenance base planning. *Management Science* **35**, 1415–1432.

The problem is formulated as a minimum cost multicommodity flow network with integral constraints, where each airplane represents a separate commodity and each arc has an upper and lower capacity of flow. Since obtaining feasible solutions from the LP relaxation is difficult, the authors propose a GRASP. See page 32.

Feo, T. and J. González-Velarde (1995). The intermodal trailer assignment problem: Models, algorithms, and heuristics. *Transportation Science* **29**, 330–341.

The problem is formulated as a set covering problem. A branch-and-bound algorithm and a GRASP are developed for solving it. The greedy strategy of the construction phase of GRASP consists in selecting at each step a feasible assignment of the most difficult to use available railcar together with the most difficult to assign trailer. To improve the constructed solution, a 2-exchange local search is applied, carrying out a complete enumeration of the solutions in the neighborhood. See also page 19.

García, J., S. Lozano, K. Smith, and F. Guerrero (2001). A comparison of GRASP and an exact method for solving a production and delivery scheduling problem. In *First International Workshop on Hybrid Intelligent Systems (HIS'01)*, Adelaide, Australia.

See also page 32.

Lourenço, H., J. Paixão, and R. Portugal (2001). Multiobjective metaheuristics for the bus-driver scheduling problem. *Transportation Sciences* **35**, 331–343.

To design a GRASP, a set N of n duties is defined and a greedy criterion based on a quantity proportional to the cost associated with the duties is used. The local search procedure uses a 1-exchange neighborhood. See also page 33.

Milidú, R., A. Pessoa, V. Braconi, E. Laber, and P. Rey (2001). Um algoritmo GRASP para o problema de transporte de derivados de petróleo em oleodutos. In *Proceedings of the XXXIII Brazilian Symposium on Operations Research*, pp. 237–246.

A GRASP is described for petroleum derivatives transportation in pipelines. The greedy function is a simple cost function obtained as sum of the total volumes of the product.

Sosnowska, D. (2000). Optimization of a simplified fleet assignment problem with metaheuristics: Simulated annealing and GRASP. In P. Pardalos (Ed.), *Approximation and complexity in numerical optimization*. Kluwer Academic Publishers.

A simulated annealing and a GRASP are proposed. In GRASP only exchanges leading to a better solution are permitted and the potentially best part of the assignment is conserved and the rest is randomly reattributed. The construction phase does not use a restricted candidate list explicitly, but a solution is built by simply trying to make the time interval between two flights as small as possible. See also page 22.

16. TELECOMMUNICATIONS

GRASP has been widely applied in the telecommunications field to problems ranging from network design to facility location and routing. Below are papers describing applications of GRASP in telecommunications.

Abello, J., P. Pardalos, and M. Resende (1999). On maximum clique problems in very large graphs. In J. Abello and J. Vitter (Eds.), *External memory algorithms and visualization*, Volume 50 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pp. 119–130. American Mathematical Society.

A semi-external memory GRASP is presented to approximately solve the maximum clique problem and maximum quasi-clique problem in very large graphs. Communities of interest are extracted from telephone call detail graphs. See also page 14.

Abello, J., M. Resende, and S. Sudarsky (2002). Massive quasi-clique detection. *Lecture Notes in Computer Science* **2286**, 598–612.

A GRASP is designed for extracting dense subgraphs on case of both nonbipartite and bipartite case. Communities of interest are extracted from telephone call detail graphs. See also page 14.

Álvarez, A., J. L. González, and K. De-Alba (2003). Scatter search for a network design problem. Technical Report PISIS-2003-02, Universidad Autónoma de Nuevo León, Facultad de Ingeniería Mecánica y Eléctrica, División de Posgrado en Ingeniería de Sistemas, Mexico.

The authors propose a Scatter Search (SS) algorithm for solving a fixed charge capacitated multicommodity network design problems on undirected networks. A GRASP-based diversification generation method (DGM) with memory features is described. In the GRASP DGM, for each commodity a certain number q of shortest paths between each origin-destination pair are kept as RCL elements. The local search consists basically in sorting the chosen paths and possibly exchange some of them in order to get a better distribution. See also page 7.

Amaldi, E., A. Capone, and F. Malucelli (2003). Planning UMTS base station location: Optimization models with power control and algorithms. *IEEE Transactions on Wireless Communications* **2**(5), 939–952.

The greedy function takes into account the fraction of traffic covered and the installation costs. Local search is a swap procedure. See also page 23.

Amaldi, E., A. Capone, F. Malucelli, and F. Signori (2003). Optimization models and algorithms for downlink UMTS radio planning. In *Wireless Communications and Networking, 2003 (WCNC 2003)*, Volume 2, pp. 827–831.

The UMTS base station location problem is addressed. Previously, in Amaldi et al. (2003), two randomized heuristics were proposed. They are here adapted for solving a similar problem. See also page 23.

Armony, M., J. Klincewicz, H. Luss, and M. Rosenwein (2000). Design of stacked self-healing rings using a genetic algorithm. *Journal of Heuristics* **6**, 85–105.

A genetic algorithm for design of stacked self-healing rings is proposed. The objective is to optimize the trade-off between the cost of connecting nodes to the ring and the cost of routing demand on multiple rings. The initial population of the genetic algorithm is made up of randomly generated solutions as well as solutions generated by a GRASP. Computational comparisons are made with a commercial integer programming package.

Brunato, M. and R. Battiti (2001). A multistart randomized greedy algorithm for traffic grooming on mesh logical topologies. Technical report, Department of Mathematics, University of Trento, Trento, Italy.

A logical topology design problem on Dense Wavelength Division Multiplexing (DWDM) optical networks is addressed. Traffic is measured at sub-wavelength resolution and the key factor to determine the fitness of a solution is the number of lightpaths required. A GRASP-like heuristic for minimizing the number of lightpaths is described. The greedy choice takes into account the load associated with each lightpath.

Canuto, S., M. Resende, and C. Ribeiro (2001). Local search with perturbation for the prize-collecting Steiner tree problems in graphs. *Networks* **38**, 50–58.

The prize collecting Steiner tree problem arises in telecommunications access network design. See page 5, page 7, and page 15.

Cooke, B., D. Kwon, D. Glotov, S. Schurr, D. Taylor, and T. Wittman (2002). Mobility management in cellular telephony. Technical report, Institute of Mathematics and its Applications, University of Minnesota, USA.

Four algorithms are proposed: a branch-and-bound, a metropolis algorithm with annealing, a genetic algorithm, and a greedy search. The latter has been derived from a GRASP proposed for the quadratic assignment problem.

Gendron, B., J.-Y. Potvin, and P. Soriano (2002). Diversification strategies in local search for a nonbifurcated network loading problem. *European Journal of Operational Research* **142**(2), 231–241.

A heuristic approach is proposed that alternates construction and local search phases. Initially, a construction method provides a feasible solution, while at subsequent construction steps, a diversification approach is adopted for exploiting information gathered along previous iterations. Two local searches are used: a pure descent search and a tabu search. An implementation of GRASP is proposed.

Klincewicz, J. (1992). Avoiding local optima in the p -hub location problem using tabu search and GRASP. *Annals of Operations Research* **40**, 283–302.

A GRASP is proposed for the p -hub location problem. Its local search procedure is based on a 2-exchange. See also page 25.

Klincewicz, J. (2002). Enumeration and search procedures for a hub location problem with economies of scale. *Annals of Operations Research* **110**, 107–122.

The greedy function of the GRASP takes into account the amount of originating and terminating traffic. The local search uses a 1 – 1 swap neighborhood structure. See also page 25.

Kumaran, K., A. Srinivasan, Q. Wang, S. Lanning, and K. Ramakrishnan (2001). Efficient algorithms for location and sizing problems in network design. In *Global Telecommunications Conference, 2001 (GLOBE-COM '01)*, Volume 4, pp. 2586–2590. IEEE.

The problems of location and sizing in network design are considered. Algorithms based on linear programming and a slightly modified GRASP are developed. In the GRASP, the construction phase is performed at random. See page 25 for details about the local search.

Li, B., F. Chen, and L. Yin (2000). Server replication and its placement for reliable multicast. In *Proceedings of the IEEE ICCCN-00*, pp. 396–401.

In a multicast network, packets are forwarded from a source (server) to group of receivers along a distribution tree, where the source is the root, the receivers are the leaves, and the multicast-capable routers are the internal nodes. The problem consists in placing multiple replicated servers within the multicast-capable routers. Several heuristics are proposed, including a GRASP. The greedy function is the router cost function, while the local search phase uses a k -exchange neighborhood structure with $k = 1$.

Liu, X., P. Pardalos, S. Rajasekaran, and M. Resende (2000). A GRASP for frequency assignment in mobile radio networks. In S. Rajasekaran, P. Pardalos, and F.Hsu (Eds.), *Mobile Networks and Computing*, Volume 52 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pp. 195–201. American Mathematical Society.

A GRASP for frequency assignment is described. The construction phase uses two greedy functions. The first chooses a vertex from the set of unselected vertices with high saturation degrees. The second function is used to assign a frequency to the selected vertex. A frequency is selected from a set of permissible frequencies that contribute little additional cost to the objective function. See also page 19.

Mahey, P. and C. Ribeiro (2002). Modeling modern multimedia traffic. *Annals of Operations Research* **110**, 107–122.

Survey of state-of-the-art algorithms for solving optimal routing problems on multi-service communication problems.

Myslek, A. (2001). Greedy randomised adaptive search procedures (GRASP) for topological design of MPLS networks. In *Proceedings of the 8th Polish Teletraffic Symposium*.

The IP/MPLS network cost optimization problem of selecting localization of nodes and links, combined with link dimensioning is addressed. A GRASP is proposed, whose greedy function uses demand flow allocation, while at each iteration of the local search phase, some selected nodes (or edges) become unavailable if provided and vice versa.

Myslek, A. and P. Karaś (2002). Heuristic methods for topological design of telecommunication networks. In *Proceedings of PGTS 2002*.

Given a list of potential node locations and a list of feasible interconnections between nodes, the generic topological network design problem consists in finding a network structure and a demand allocation pattern that minimizes the cost of the network. A pool of heuristics are proposed for solving the problem, including a Simulated Allocation (SAL) and a hybrid GRASP that uses SAL as local search.

Oliveira, C. and F. Gomes (1999). Two metaheuristics for channel allocation in mobile telephony. Technical report, Artificial Intelligence Laboratory, Universidade Federal do Ceará, Fortaleza, Brazil.

Two heuristics are proposed: GRASP and Asynchronous Team (A-Team). The construction phase of the proposed GRASP is realized by a procedure that at each step chooses the next antenna to which a frequency will be assigned. In the RCL construction, priority is given to transmitters with fewer options of frequency assignment. To implement the local search phase, a down hill algorithm is used. It performs random perturbations in

the solution, exchanging the frequency of one antenna by another randomly chosen. See also page 20.

- Pasillio, E. (1998). A greedy randomized adaptive search procedure for the multi-criteria radio link frequency assignment problem. Technical report, Department of ISE, University of Florida, Gainesville, FL 32611-6595.

The objective of the problem addressed here is to minimize the order and the span of the solution set. The local search procedure attempts to eliminate each channel from the communication network. See also page 21.

- Poppe, F., M. Pickavet, P. Arijs, and P. Demeester (1997). Design techniques for SDH mesh-restorable networks. In *Proceedings of the European Conference on Networks and Optical Communications (NOC'97), Volume 2: Core and ATM Networks*, pp. 94–101.

To design low cost reliable telecommunication networks, three algorithms are proposed: an integer linear programming algorithm (branch-and-cut-and-price), a GRASP, and a zoom-in approach that combines a genetic algorithm with deterministic optimization routines. The greedy choice of the proposed GRASP is to favor paths having lowest additional cost. The local search iteratively tries to reroute some paths.

- Prais, M. and C. Ribeiro (2000). Reactive GRASP: An application to a matrix decomposition problem in TDMA traffic assignment. *INFORMS Journal on Computing* **12**, 164–176.

A geostationary communication satellite has a number of spot beam antennas covering geographically distributed areas. According to the slot switching configuration on the on-board switch, the uplink traffic received at the satellite has to be immediately sent to ground areas through a set of transponders. The slot switching configurations are determined through the solution of a time slot assignment problem, which is equivalent to the decomposition of a nonnegative traffic matrix into the sum of a family of switching mode matrices. A Reactive GRASP is proposed. See also page 5 and page 21.

- Resende, L. and M. Resende (1999). A GRASP for frame relay PVC routing. In *Proc. of the Third Metaheuristics International Conference*, pp. 397–402.

The objective of the problem solved here is to minimize PVC delays while balancing trunk loads. The local search procedure reroutes each PVC, one at a time, checking each time if the new route taken together with the remaining fixed routes improves the objective function. See also page 30.

- Resende, M. (1998). Computing approximate solutions of the maximum covering problem using GRASP. *Journal of Heuristics* **4**, 161–171.

A GRASP for maximum covering is proposed. Maximum covering problems arise in telecommunications network location applications. See also page 28.

- Resende, M. and C. Ribeiro (2003). GRASP with path-relinking for private virtual circuit routing. *Networks* **41**, 104–114.

Variants of a GRASP with path-relinking heuristic are proposed for the offline PVC routing problem. See also page 9 and page 30.

- Resende, M. and O. Ulular (1997). SMART: A tool for AT&T Worldnet access design – Location of Cascade 9000 concentrators. Technical report, AT&T Labs Research, Florham Park, NJ 07932 USA.

This report describes SMART, a software tool for finding low cost configurations of Cascade 9000 concentrators in the AT&T Worldnet backbone access network. The concentrator location problem is stated and cost model is presented for concentrator configurations. This cost model is used in a GRASP, proposed for finding approximate solutions to the concentrator location problem. The greedy choice favors the points-of-presence (POPs) with smallest incremental cost. The local search implements a simple 2-exchange.

Ribeiro, C. and I. Rosseti (2002). A parallel GRASP for the 2-path network design problem. *Lecture Notes in Computer Science* **2004**, 922–926.

A parallel GRASP with path-relinking is proposed for solving the 2-path network design problem. See also page 13 and page 18.

Rosseti, I. (2003). *Heurísticas para o problema de síntese de redes a 2-caminhos*. Ph. D. thesis, Department of Computer Science, Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil.

The 2-path network design problem (2PNDP) consists in finding a minimum weighted subset of edges containing a 2-path between the endpoints of every origin-destination in D , where a 2-path between the pair $(s, t) \in D$ is a sequence of at most two edges connecting s to t . To solve 2PNDP, sequential and parallel heuristics are proposed, included variants and combinations of GRASP. In Portuguese. See also page 18.

Srinivasan, A., K. Ramakrishnan, K. Kumaram, M. Aravamudam, and S. Naqvi (2000). Optimal design of signaling networks for Internet telephony. In *IEEE INFOCOM 2000*.

An approach for efficient design of a signaling network for a network of software switches supporting Internet telephony is proposed. Optimal load balancing for given demand forecast is formulated as a quadratic assignment problem, which is solved with a GRASP.

Vieira, C. and P. Gondim (2001). Uma nova estratégia para aplicação do GRASP ao problema de alocação de canal. Technical Report 070/DE9/01, Departamento de Engenharia de Sistemas, Instituto Militar de Engenharia, Rio de Janeiro, Brazil.

A special channel allocation problem arising in telecommunications is addressed. See page 22.

Xu, J. and S. Chiu (1996). Solving a real-world field technician scheduling problem. In *Proceedings of the International Conference on Management Science and the Economic Development of China*, pp. 240–248.

The local search implements four different moves, among them the 2-exchange and a swap that exchanges an assigned job with another job unassigned under the candidate schedule. See page 33.

Xu, J. and S. Chiu (2001). Effective heuristic procedure for a field technician scheduling problem. *Journal of Heuristics* **7**, 495–509.

The objective of the field technician scheduling problem is to assign a set of jobs at different locations with time windows to technicians with different job skills. Several heuristics are designed and tested for solving the problem: a pure greedy heuristic, a GRASP, and a local search algorithm. The greedy choice of the GRASP proposed is to select jobs with the highest unit weight. The local search implements four different moves, among them the 2-exchange and a swap that exchanges an assigned job with another job unassigned under the candidate schedule. See also page 33.

17. ELECTRICAL POWER SYSTEMS

GRASP has been applied to problems arising in planning and operations of electrical power systems. The following papers exemplify these applications.

Bahiense, L., G. Oliveira, and M. Pereira (2001). A mixed integer disjunctive model for transmission network expansion. *IEEE Transactions on Power Systems* **16**, 560–565.

Given the nonconvex nature of the transmission network expansion problem, its classical nonlinear mixed integer formulation does not guarantee an optimal solution. An alternative mixed integer linear disjunctive formulation is proposed. The mixed integer program is solved by a commercial branch and bound code, where the upper bound in

the bounding phase is obtained by applying the reactive GRASP proposed in Binato et al. (2001).

- Binato, S. and G. Oliveira (2002). A Reactive GRASP for transmission network expansion planning. In C. Ribeiro and P. Hansen (Eds.), *Essays and surveys in metaheuristics*, pp. 81–100. Kluwer Academic Publishers.

The GRASP previously proposed by Binato, Oliveira, and Araújo (1998) for the transmission network expansion problem is enhanced with the reactive scheme of Prais and Ribeiro (2000). A bias distribution function of Bresina (1996) is applied to bias the random greedy construction phase towards the most promising variables. See also page 5.

- Binato, S., G. Oliveira, and J. Araújo (2001). A greedy randomized adaptive search procedure for transmission expansion planning. *IEEE Transactions on Power Systems* **16**, 247–253.

A GRASP for a long term transmission expansion planning problem is proposed. The greedy function minimizes the load curtailment required to eliminate all operational violations. The local search phase is based on circuit exchanges.

- Faria Jr., H., S. Binato, M. Resende, and D. Falcão (2004). Power transmission network design by greedy randomized adaptive path relinking. *IEEE Transactions on Power Systems*. To appear.

A new metaheuristic based on ideas in GRASP and path relinking is applied to solve static power transmission network design problems. See also page 8.

- Viana, A., J. de Sousa, and M. Matos (2003). Using GRASP to solve the unit commitment problem. *Annals of Operations Research* **120**, 117–132.

The unit commitment problem consists in deciding, over a given planning horizon, the set of electric generators to be committed and defining the production levels required for each generator so that load and spinning reserve requirements are verified at a minimum production cost. The GRASP construction phase proposed applies a greedy criterion based on fuel cost, start-up cost, and shut-down cost. The local search procedure uses a 1-flip neighborhood, where neighbors of a given solution are obtained by changing the current status of a single unit.

- Vogel, I., M.-L. Flottes, and C. Landrault (2002). Initialisation des circuits séquentiels avant test intégré et scan partiel. Technical report, Laboratoire d’Informatique, de Robotique et de Micro-électronique de Montpellier, Montpellier, France.

The GRASP proposed by Pardalos, Qian, and Resende for the feedback set problem is applied for solving the sequential circuits initialization problem. In French.

18. BIOLOGY

Recent work in computational biology has applied the concepts of GRASP. The papers below illustrate this.

- Andreatta, A. and C. Ribeiro (2002). Heuristics for the phylogeny problem. *Journal of Heuristics* **8**, 429–447.

A phylogeny is an evolutionary tree that relates taxonomic units, based on their similarity over a set of characters. To solve the phylogeny problem is to find a phylogeny with the minimum number of evolutionary steps, i.e. applying the so-called parsimony criterion. Several heuristic approaches are studied and tested, including a GRASP. Three different neighborhood structures are investigated: nearest neighborhood interchange, the single step neighborhood, and subtree pruning and regrafting.

- Andronesco, M. and B. Rastegari (2003). Motif-GRASP and Motif-ILS: Two new stochastic local search algorithms for motif finding. Technical report, Computer Science Department, University of British Columbia, Vancouver, Canada.

A motif is a conserved pattern thought to exist in several biosequences such as DNA, RNA, and proteins. Given N biosequences S_i , $i = 1, 2, \dots, N$ with length n_i and a number L , the problem of motif finding consists in finding a sequence M_i of length L for each biosequence such that their similarity grade is maximized. A candidate solution is represented as a set a_1, a_2, \dots, a_N , where $a_k \in [1, n_k - L + 1]$, for each $k \in [1, N]$. All candidate solutions correspond to all possible combinations of a_i assignment. Several greedy functions are proposed based on the a weight defined on the starting point of the motif. The neighborhood structure used in the local search procedure is a 1-exchange.

Brown, D. (2000). *Algorithmic methods in genetic mapping*. Ph. D. thesis, Cornell University, Ithaca, NY, USA.

A survey of existing methods for genetic mapping problems is presented and several new algorithms, including a GRASP, proposed. The greedy function is defined on bin length, while the local search first removes from the sample those population members that do not affect on the objective function value.

Brown, D., T. Vision, and S. Tanksley (2000). Selecting mapping: A discrete optimization approach to select a population subset for use in a high-density genetic mapping project. *Genetics* **155**, 407–420.

A GRASP is proposed for selecting a population subset for use in a high-density genetic mapping project. At each iteration of the construction phase, one among the r unchosen population members which most improve the objective function value is added to the solution. Very small sized RCLs (i.e. $r = 3$ and $r = 5$) are used. The implemented local search removes from the current solution some members and greedily includes other members.

Fried, C., W. Hordijk, S. Prohaska, C. Stradler, and P. Stradler (2003). The footprint sorting problem. Technical report, Bioinformatics, Department of Computer Science, University of Leipzig, Germany.

Phylogenetic footprints are short pieces of no-coding DNA sequence in genes that are conserved between evolutionary distant species. It is shown that solving the footprint sorting problem requires the solution of a minimum weight vertex feedback set problem. For this the GRASP of Festa et al. (2001) is used.

Iorvik, V., E. Triantaphyllou, T. Liao, and S. Waly (1999). Predicting muscle fatigue via electromyography: A comparative study. In *Proceedings of the 25th International Conference on Computers and Industrial Engineering*, pp. 277–280.

A comparison of some state-of-the-art AI predictive and statistical techniques, including a GRASP, is presented.

Krasnogor, N., D. Pelta, W. Russo, and G. Terrazas (1998). A GRASP approach to the protein structure prediction problem. Technical report, LIFIA Lab, University of La Plata, La Plata, Argentina.

The applicability of a GRASP for solving a special protein folding problem is presented. The goal is to predict from the molecular sequence of a given protein its particular 3D structure.

Reynolds, A., J. Dicks, I. Roberts, J. Wesselink, B. de la Iglesia, V. Robert, T. Boekhout, and V. Rayward-Smith (2003). Algorithms for identification key generation and optimization with application to yeast identification. In *Applications of Evolutionary Computing*, Volume 2611 of *Lecture Notes in Computer Science*, pp. 107–118. Springer-Verlag.

For the automated creation of low cost identification keys, several algorithms are described. One of them applies the greedy randomized strategy of the GRASP framework.

Ribeiro, C. and D. Vianna (2003). A GRASP/VND heuristic for the phylogeny problem using a new neighborhood structure. Technical report, Department of Computer Science, Catholic U. of Rio de Janeiro, Rio de Janeiro, Brazil.

The phylogeny problem consists in finding a phylogeny with the minimum number of evolutionary steps, where a phylogeny is a tree that relates taxonomic units based on their similarity over a set of characters. The authors propose a hybridization of GRASP and VND. See also page 9.

19. VLSI DESIGN

GRASP has been used to solve circuit partitioning problems, as illustrated by the following papers.

Areibi, S. (1999). GRASP: An effective constructive technique for VLSI circuit partitioning. In *Proc. IEEE Canadian Conference on Electrical & Computer Engineering (CCECE'99)*, Volume 1, pp. 462–467.

A GRASP is proposed to obtain good initial solutions for an iterative improvement technique. See page 23 and page 26.

Areibi, S., M. Moussa, and H. Abdullah (2001). A comparison of genetic/memetic algorithms and other heuristic search techniques. In *Proceedings of IC-AI 2001*.

Several constructive procedures for circuit partitioning problems are compared, including a genetic algorithm, a memetic algorithm, and a GRASP.

Areibi, S. and A. Vannelli (1997). A GRASP clustering technique for circuit partitioning. In J. Gu and P. Pardalos (Eds.), *Satisfiability problems*, Volume 35 of *DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, pp. 711–724. American Mathematical Society.

The number of clusters is predetermined as a function of the number of partitions required. Initially, the heuristic reads the circuit description and resizes the blocks to be used by GRASP, which utilizes only the construction phase to generate the number of required clusters. The GRASP construction phase is followed by a post-processing stage, in which a simple dynamic hill climbing algorithm is used as local search to improve the initial solution generated. See also page 23.

Areibi, S. and A. Vannelli (2000). Efficient hybrid search techniques for circuit partitioning. In *IEEE 4th World Multiconference on Circuits, Systems, Communications & Computers*.

See page 7, page 23, and page 26.

20. AUTOMATIC DRAWING

GRASP has been used to find approximate solutions to problems related to automatic drawing. This section lists these papers.

Binucci, C., W. Didimo, G. Liotta, and M. Nonato (2002). Labeling heuristics for orthogonal drawings. In *Proceedings of GD'98 – Symposium on Graph Drawing*, Volume 2265 of *Lecture Notes in Computer Science*, pp. 139–153. Springer-Verlag.

Several heuristics (including a GRASP) for computing an orthogonal drawing of a graph with labels are implemented and compared.

Fernández, E. and R. Martí (1999). GRASP for seam drawing in mosaicking of aerial photographic maps. *Journal of Heuristics* **5**, 181–197.

Commercial aerial photographic maps are often so large that it is necessary to produce one map from two or even more photographs. These are combined, two at a time, in a process called *mosaicking*. The most difficult step in the mosaicking process is *seam-drawing*. A GRASP is proposed for solving the seam-drawing process.

Laguna, M. and R. Martí (1999). GRASP and path relinking for 2-layer straight line crossing minimization. *INFORMS Journal on Computing* **11**, 44–52.

A GRASP with path-relinking is developed for the problem of minimizing straight line crossings in a 2-layer graph. The greedy criterion of the construction phase is based on the degree of the vertices and a value based restricted candidate list is used. Each step of the improvement phase consists in selecting each vertex to be considered for a move. A probabilistic selection rule is used such that vertices with high degree are more likely to be selected first at each step of this process. See also page 5 and page 8.

Martí, R. and V. Estruch (2001). Incremental bipartite drawing problem. *Computers and Operations Research* **28**, 1287–1298.

A GRASP is proposed for the incremental arc crossing minimization problem for bipartite graphs. Computational experiments are done on 450 instances and results are compared with a branch and bound algorithm. See also page 16.

Martí, R. and M. Laguna (2003). Heuristics and meta-heuristics for 2-layer straight line crossing minimization. *Discrete Applied Mathematics* **127**, 665–678.

Extensive computational results are presented using 12 heuristics and two meta-heuristics for the 2-layer straight line crossing minimization problem. On dense graphs, a tabu search meta-heuristic does best with GRASP a close second. On low-density graphs, GRASP outperforms all other approaches. See also page 16.

Ng, K. and B. Trifonov (2003). Automatic bounding volume hierarchy generation using stochastic search methods. In *CPSC532D Mini-Workshop "Stochastic Search Algorithms"*.

A bounding volume hierarchy is used for improving the efficiency of ray tracing based rendering. Finding good hierarchies is difficult, since the number of hierarchies grows exponentially with the number of scene objects. A GRASP is designed for improving previously proposed heuristics. The greedy function used is based on subdivision points, while local search is basically a perturbation procedure.

Osman, I., B. Al-Ayoubi, and M. Barake (2003). A greedy random adaptive search procedure for the maximal planar graph problem. *Computers and Industrial Engineering* **45**, 635–651.

A GRASP is proposed and tested for the weighted maximal planar graph problem. The construction is a randomized version of the Green and Al-Hakim algorithm (1985). A new data structure is introduced, reducing the complexity of the construction from $O(n^3)$ to $O(n^2)$. Local search uses four types of moves proposed by Pesch, Glover, Bartsch, Salewski, and Osman (1995). See also page 17.

Osman, I., B. Al-Ayoubi, M. Barake, and M. Hasan (2000). A greedy random adaptive search procedure for the weighted maximal planar graph problem. Technical report, School of Business and Center for Advanced Mathematical Sciences, American University of Beirut, Beirut, Lebanon.

A GRASP is proposed and tested for the weighted maximal planar graph problem. See also page 17.

Osman, I., M. Hasan, and A. Abdullah (2002). Linear programming based meta-heuristics for the weighted maximal planar graph. *Journal of the Operational Research Society* **53**, 1142–1149.

Two meta-heuristics are described, both derived from an ILP relaxation. The first one takes into account only variables with fractional value greater than half in the ILP relaxation to build an initial subgraph from which a planar subgraph is extracted with the help of a GRASP and triangulation of faces. The second approach considers only edges having integer value in the ILP relaxation, while the remaining edges are sorted in descending order of their weights. Those edges that do not violate a planarity test are thus candidate for insertion to obtain a feasible solution using GRASP. See also page 17.

Resende, M. and C. Ribeiro (1997). A GRASP for graph planarization. *Networks* **29**, 173–189.

A GRASP is described that extends the two-phase heuristic of Goldschmidt and Takvorian (*Networks*, v. 24, pp. 69–73, 1994). Computational experience on a large set of

standard test problems is presented. On almost all test problems considered, the heuristic either matches or finds a better solution than previously described graph planarization heuristics. In several cases, previously unknown optimal solutions are found. See also page 18.

Ribeiro, C. and M. Resende (1999). Algorithm 797: Fortran subroutines for approximate solution of graph planarization problems using GRASP. *ACM Transactions on Mathematical Software* **25**, 341–352.

A set of Fortran subroutines that implements the GRASP for graph planarization of Resende and Ribeiro (1997) is presented. See also page 11 and page 18.

21. MISCELLANEOUS

The papers in this section could not be categorized into any of the previous section in this paper.

de Noronha, T. F. (2001). Algoritmos e estratégias de solução para o problema do gerenciamento de sondas de produção terrestre na bacia petrolífera potiguar. *Revista Eletrônica de Iniciação Científica* **1**.

Several different heuristics are proposed, including a GRASP, for a petroleum production planning problem. The greedy function is a simple cost function related to the production, while the local search phase looks for improving solutions by swapping paths. In Portuguese.

de Souza, C. C., C. B. Medeiros, and R. S. Pereira (1996). Integrating heuristics and spatial databases: A case study. Technical Report IC-96-18, Institute of Computing, Universidade Estadual de Campinas, Campinas, Brazil.

Part of the ongoing efforts at IC-UNICAMP to apply heuristic algorithms to vectorial georeferenced data to help decision support in urban planning is described. A first prototype, implemented in C++, and tested on support planning activities for the São Paulo State Post Office System in Brazil is presented. The problem is a special partition problem, where the number of clusters in the partition (number of districts in the distribution zone) must be minimized. The problem is represented by building a special undirected graph that has two main characteristics: connectivity and information about the mailman daily loads. To solve the problem, a set of randomized heuristics, including a GRASP, are proposed.

Demirer, R. and B. Eksioğlu (1998). Subset selection in multiple linear regression: A new mathematical programming approach. Technical Report School of Business working paper no. 284, School of Business, University of Kansas, Lawrence, Kansas, USA.

A new mathematical programming model is proposed. It is parametrically solved to obtain a collection of efficient subsets. The parametric solution requires repeatedly solving a mathematical program which is done with either a Lagrangian relaxation based heuristic or a GRASP.

Festa, P. and G. Raiconi (2001). GRASP in switching input optimal control synthesis. In *Proceedings of MIC'2001*, pp. 381–385.

Several optimal control problems are introduced. A GRASP is designed. In the construction phase, the greedy criterion minimizes the quadratic costs in the Riccati equation. The neighborhood structure used in the local search phase is defined on the Hamming distance.

Fraçois, H. and O. Boëffard (2002). The greedy algorithm and its application to the construction of a continuous speech database. In *Proceedings of LREC-2002*, Volume 5, pp. 1420–1426.

A general framework of the construction of databases characterized by different linguistic features is addressed. A small sized continuous speech database is needed, at the same time based on a maximum number of phonetic units. After recorded, the set of sentences

constitutes the source from which the text-to-speech synthesizer draws the needed acoustic units. The problem is to find the smallest subset of sentences that covers all needed units. A greedy algorithm is described to solve the problem and the development of a GRASP is proposed as future work.

Gandibleux, X., D. Vancoppenolle, and D. Tuyttens (1998). A first making use of GRASP for solving MOCO problems. Technical report, University of Valenciennes, France.

An extension of GRASP, to solve multi-objective combinatorial optimization (MOCO) problems, is considered. In particular, classical covering, assignment, knapsack, and scheduling problems with multiple objectives are used as benchmarks. Computational results compare GRASP solutions for a benchmark set of test problems and results are discussed in comparison with an exact method, when available. In French.

Ghosh, J. (1996). Computational aspects of the maximum diversity problem. *Operations Research Letters* **19**, 175–181.

Two variations of the maximum diversity problem are addressed. This problem arises when m elements are to be selected from an n -element population based on inter-element distances. Using a reduction from the vertex cover problem, a GRASP is proposed.

Gomes, A. and J. Oliveira (2001). A GRASP approach to the nesting problem. In *Proceedings of MIC'2001*, pp. 47–52.

A GRASP is proposed to solve a variant of the classical nesting problem, whose objective is to minimize the length of a single plate used to produce a given set of smaller pieces. Two greedy criteria are tested. The first one is the layout length, measured as the maximum coordinate in the current layout, while the second one is the added internal waste, measured as the potential area lost when placing one piece. The local search phase uses neighbors obtained by exchanging pairs of pieces in the sequence output of the construction phase.

Juillé, H. and J. Pollack (1998). A sampling-based heuristic for tree search applied to grammar induction. In *Proceedings of the Fifteenth National Conference on Artificial Intelligence*.

SAGE, a new search algorithm that incorporates the same fundamental mechanisms as the most popular metaheuristics, is presented. It is an iterative search procedure that at each iteration performs a construction phase and a competition phase. In the construction phase, SAGE implements a set of elementary randomized searches and a meta-level heuristic that controls the search procedure by distributing the alternatives among the searches. Scope of the competition phase is to favor the most promising search alternatives.

Locatelli, M. (1998). A class of heuristic methods for the maximization of the l_1 -norm over parallelotopes. Technical report, Dipartimento di Sistemi ed Informatica, Università di Firenze, Firenze, Italy.

For the maximization of the l_1 -norm over parallelotopes, a class of heuristics is proposed that includes a slightly modified GRASP, in which between a greedy construction and local search phases a filter phase is inserted to avoid performing local search from bad starting solutions. The local search uses is a variant of a 1-flip neighborhood.

Medeiros, M., M. Resende, and A. Veiga (2001). Piecewise linear time series estimation with GRASP. *Computational Optimization and Applications* **19**, 127–144.

A GRASP is proposed to build piecewise linear statistical models with multivariate thresholds. The construction phase consists of sequentially choosing hyperplanes until the maximum number of hyperplanes is reached. The greedy function orders the possible hyperplanes with respect to the sum of squared errors of the fitted data. The local search is a 2-exchange heuristic.

Medeiros, M., A. Veiga, and M. Resende (2002). A combinatorial approach to piecewise linear time analysis. *Journal of Computational and Graphical Statistics* **11**, 236–258.

A new approach to modeling threshold processes is proposed. It is based on a linear model with time-varying parameters. This formulation is shown to be closely related to the self-exciting threshold autoregressive models (SETAR) with the advantage that it incorporates linear multivariate thresholds. A GRASP is proposed to estimate the parameters of the model. The greedy choice takes into account the sum of squared errors of the fitted data. The local search is a 2-exchange heuristic.

Mockus, J., E. Eddy, A. Mockus, L. Mockus, and G. Reklaitis (1997). *Bayesian discrete and global optimization*. Kluwer Academic Publishers.

This book describes the Bayesian approach to discrete optimization. A Bayesian heuristic algorithm version of GRASP is described.

Neto, T. and J. Pedroso (2001). GRASP for linear integer programming. In *Proceedings of MIC'2001*, pp. 377–380.

The GRASP framework is extended for solving general linear integer problems. The key is to split the variables into a set of integer and a set of linear variables. Then, GRASP finds values of the integer variables that are replaced in the original problem, which becomes a pure continuous problem solvable by any linear programming solver.

Neto, T. and J. Pedroso (2003). GRASP for linear integer programming. In M. Resende and J. de Sousa (Eds.), *Metaheuristics: Computer decision-making*, pp. 545–574. Kluwer Academic Publishers.

A preliminary version in this paper appeared in Neto and Pedroso (2001). Here, the GRASP framework is extended for solving general linear integer problems. The key is to split the variables into a set of integer and a set of linear variables. Then, GRASP finds values of the integer variables that are replaced in the original problem, which becomes a pure continuous problem, solvable by any linear programming solver.

Palubeckis, G. and A. Tomkevicius (2002). GRASP implementations for the unconstrained binary quadratic optimization problem. *Information Technology and Control* **24**, 14–20.

A classical GRASP framework and an enhanced GRASP that uses a simple tabu search as local search are proposed. Numerical results show that the enhancement introduced in the classical GRASP implementation produces higher quality solutions. See also page 9.

Roli, A. and M. Milano (2002). MAGMA: A multiagent architecture for metaheuristics. Technical Report DEIS-LIA-02-007, DEIS, Università degli Studi di Bologna, Bologna, Italy.

The main metaheuristic schemes, including GRASP, are revisited in a multiagent perspective and a uniform framework called MAGMA is provided.

Sellmann, M. and W. Harvey (2002). Heuristic constraint propagation using local search for incomplete pruning and domain filtering of redundant constraints for the social golfer problem. In *CPAIOR 2002*.

It is hard to efficiently establish the existence of a feasible solution in a constraint satisfaction problem. The most common approach is to implicitly explore the feasible region. This paper proposes to add tight redundant constraints, possibly hard to be verified exactly, but that can be checked by applying heuristics. One heuristic strategy considered follows a GRASP strategy.

Silva, G., L. Ochi, and S. Martins (2003). Experimental comparison of greedy randomized adaptive search procedures for the maximum diversity problem. Technical report, Department of Computer Science, Federal Fluminense University, Niterói, Brazil.

Given a large collection C of elements, the maximum diversity problem consists in finding optimally diverse subsets of C . Ghosh (1996) proposed a GRASP for approaching this problem. Here, a new GRASP is depicted, whose construction phase implements three different strategies based on distance greedy function. The local search phase uses

two neighborhood structures. One is the structure defined by Ghosh, while the second one is a 2-exchange neighborhood.

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