ator $\mathcal{W} = \mathcal{W}(x,y,X,Y)$. We use the same notation as in the proof of Theorem 1. From the hypotheses, eq. (1), and the fact that in each term $x^j y^k X^l Y^m$ of $\mathcal{W}$, $j+k=l+m$, it follows that $\mathcal{W}$ is invariant under M^+, J^+, and

$$T_1 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & & 1 \\ & & 1 & \end{pmatrix}, \quad T_2 = \begin{pmatrix} & & 1 & \\ & & & 1 \\ 1 & & & \\ & 1 & & \end{pmatrix}, \quad T_3 = \begin{pmatrix} \omega & & & \\ & \omega & & \\ & & \omega^{-1} & \\ & & & \omega^{-1} \end{pmatrix}.$$

M^+, J^+, T_1 generate a group of order 16192 consisting of the matrices $\begin{pmatrix} A & \\ & BA \end{pmatrix}$, $A \in \mathfrak{G}_{192}$, $B \in \mathfrak{H}_{16}$, where $\mathfrak{H}_{16} = \{\delta\begin{pmatrix} 1 & \\ & \pm 1\end{pmatrix}, \delta\begin{pmatrix} & 1 \\ \pm 1 & \end{pmatrix} : \delta \in \{1, i, -1, -i\}\}$ is a normal subgroup of $\mathfrak{G}_{192}$; and $\mathfrak{G}_{192} = \bigcup_{k=1}^{12} A_k \mathfrak{H}_{16}$, where $A_1, ..., A_6$ are

$$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}, \begin{pmatrix} 1 & \\ & i \end{pmatrix}, 2^{-1/2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, 2^{-1/2}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}, 2^{-1/2}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}, 2^{-1/2}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix},$$

and $A_{6+j} = rA_j$, $1 \le j \le 6$. Then M^+, J^+, T_1, T_2, T_3 generate a group $\mathfrak{G}$ of order $6144p$ consisting of the matrices

$$\begin{pmatrix} \omega^\nu A & \\ & \omega^{-\nu} BA \end{pmatrix}, \quad \begin{pmatrix} & \omega^\nu A \\ \omega^{-\nu} BA & \end{pmatrix}, \qquad 0 \le \nu \le p-1,$$
$$A \in \mathfrak{G}_{192}, \quad B \in \mathfrak{H}_{16}.$$

Now $\mathcal{W}$ is invariant under $\mathfrak{G}$. Let $\mathcal{M}$ be the set of all invarinats of $\mathfrak{G}$. Clearly $\mathcal{M}$ contains $\mathcal{N} = \mathbf{C}[\eta_8, \theta_{16}, \gamma_{24}]$. To show $\mathcal{M} = \mathcal{N}$, we define a_d, b_d as before and will show $a_d = b_d$ for all d. We have

$$\sum_0^\infty b_d \lambda^d = 1/(1-\lambda^8)(1-\lambda^{16})(1-\lambda^{24}).$$

From (2), for all $p > d$, a_d is the coefficient of λ^d in

$$\frac{1}{6144p} \sum_{\nu,A,B} \left\{ \frac{1}{|I-\lambda\omega^\nu A|\,|I-\lambda\omega^{-\nu}BA|} + \frac{1}{|I-\lambda^2 ABA|} \right\} = \Sigma_{\mathrm{I}} + \Sigma_{\mathrm{II}} \text{ say.}$$

In Σ_{I} we put $A = A_k B'$:

$$\Sigma_{\mathrm{I}} = \frac{1}{24p} \sum_{k=1}^{12} \sum_{\nu=0}^{p-1} f(A_k; \lambda\omega^\nu) f(A_k; \lambda\omega^{-\nu}),$$