A New Class of Multiple Description Scalar Quantizer and Its Application to Image Coding

Chao Tian, *Student Member, IEEE,* and Sheila S. Hemami, *Senior Member, IEEE*

***Abstract*—We introduce a new class of multiple description scalar quantizer (MDSQ), which will be referred to as the modified MDSQ (MMDSQ). The structure of MMDSQ is fundamentally different from the MDSQ proposed by Vaishampayan. Analysis shows that at high rates for sources with smooth pdfs, MMDSQ achieves the same performance as entropy-constrained MDSQ using a uniform central quantizer. Compared with MDSQ, MMDSQ features a more efficient central-side distortion tradeoff control mechanism, without requiring the design and implementation of a complicated index assignment scheme. The simple and efficient implementation of MMDSQ makes it suitable for image coding: It can be naturally incorporated into existing coders to form new multiple description coders. Such a system is constructed using an existing image coder, and the performance of the resulting system is compared to that of the same coder but using Vaishampayan's MDSQ. Simulation results on natural images reveal that the system based on MMDSQ outperforms that based on MDSQ.**

***Index Terms*—Image compression, multiple descriptions, scalar quantization.**

I. Introduction

THE multiple description (MD) problem poses the following question: When a source is encoded into two descriptions at the some rates, what are the best reconstructions using each of the two descriptions as well as that of their joint description? This fundamental problem in source coding is applicable to situations when two unreliable channels are present between the transmitter and the receiver as well as in packet networks with loss (see [1] for an excellent review). An example application is image transmission over wireless channels.

The MD problem is most often considered when the rates of the two descriptions are the same, and the distortions generated by each individual description are equal, which is called *the balanced case*; this letter considers only this case. The distortion generated by the joint description is the *central distortion*, while the distortions generated by the individual descriptions are the *side distortions*. The central and side distortions cannot both be low, and achieving different tradeoffs between them is a challenge in designing any MD system.

One approach—MD scalar quantization (MDSQ)—was first proposed by Vaishampayan [2], [3] as a practical solution to the MD problem, and its analysis was recently thoroughly addressed in [4]. One desirable property of MDSQ is that it provides an asymptotic performance close to the rate-distortion bound. In this letter, we introduce a new class of MDSQ, namely the Modified MDSQ (MMDSQ), which not only has this property but also features simple implementations.

Manuscript received September 16, 2004; revised November 20, 2004. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Jean-Christophe Pesquet.

The authors are with the School of Electrical and Computer Engineering, Cornell University, Ithaca, NY 14853 USA (e-mail: ctian@ece.cornell.edu; hemami@ece.cornell.edu).

Digital Object Identifier 10.1109/LSP.2005.843764

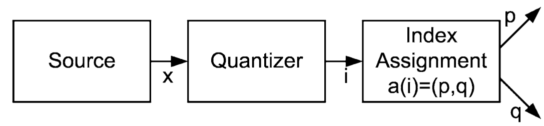

Fig. 1. Multiple description encoder diagram.

II. MMDSQ: Analysis and Design

A. MDSQ and Its Analysis

The idea behind MDSQ is to create two coarse side quantizers that produce acceptable side distortions when used alone; the two coarse side quantizers are combined to produce a finer central quantizer. The side quantizers in MDSQ are not conventional quantizers: Rather than consisting of a continuous interval, a side quantizer cell in MDSQ is usually the union of several noncontiguous intervals. MDSQ mainly consists of a central quantization step and an index assignment step, as depicted in Fig. 1. The index assignment is often represented by an index assignment matrix, whose elements are the (single) input index, while the column and row indices are the output as an index pair (see [2] for more details). The design of the optimal index assignment is still an open problem, though heuristic methods to design good index assignments were given in [2].

We use the mean squared error as the error measure. To bound the asymptotic performance, the Shannon lower bound for the MD problem [6] will be used, which is tight at high resolution. Through some algebra, it can be shown that at high resolution, if the side distortion is given by $D_1 = bP_x 2^{-2(1-\eta)R}$ for real numbers $b \geq 0, 0 < \eta < 1$, then the central distortion is given by $D_0 \geq \frac{P_x}{4b} 2^{-2(1+\eta)R}$, where R is the rate of each description, $P_x = (2\pi e)^{-1} 2^{2H_p}$, and H_p is the differential entropy of the source. Note that the product of the central and side distortions is bounded by a constant at a fixed rate, which suggests there is a performance *tradeoff* between the central and side quantizers. This product is, therefore, used as the information theoretic bound for the quantizers.

The asymptotic analysis in [7] shows that the distortion product of the central and side distortions for entropy-constrained MDSQ with a uniform central quantizer is

$$D_{Q1} D_{Q0} \approx \left(\frac{2\pi e}{12}\right)^2 \frac{P_x^2}{4} 2^{-4R} \quad (1)$$

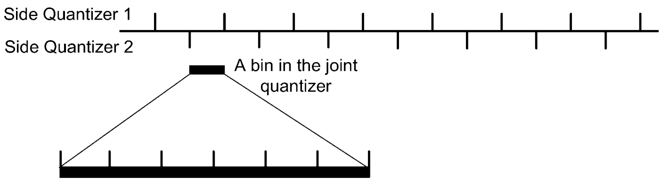

Fig. 2. Structure of MMDSQ.

which is 3.07 dB away from the rate-distortion bound. It was later shown that this product can be reduced by 0.4 dB when nonuniform central quantizers are used [4].

B. Modified MDSQ

1) Structure of MMDSQ: MMDSQ is a two-stage system. The first stage consists of two uniform side quantizers with staggered bins. The joint quantizer formed by these two staggered quantizers has a smaller half-size bin. In the second stage, each bin in the joint quantizer from the first stage is divided uniformly into (a fixed number of) finer bins to form the second stage quantizer (see Fig. 2).

We form the two descriptions as follows: The quantization indices from each side quantizer are entropy coded into the bitstream of each description; then, the indices from the second stage are again entropy coded but split evenly between each description. The receiver decodes the descriptions as follows: When only one description is received, it decodes the first stage information using the corresponding side quantizer; if both descriptions are available, it decodes the first stage information and uses the joint quantizer of the two side quantizers to narrow the possible source sample range and then decodes the second stage information using the second stage quantizer.

Such a two-stage structure is not totally new: El Gamal and Cover [8] used an inherent two-stage approach to derive an achievable rate region for the MD problem. Another closely related work is [9], where dithered quantizers are formulated in a similar two-stage structure. Their discussion focused on independent dithering in the first stage, which generates an additional 0.5-bit loss in the sum rate for scalar quantizers; dependent dithers (which correspond to staggering) as a possible improvement were discussed briefly but in respect to the ratio of (D_1/D_0), rather than the overall performance. It can be shown, however, that with such a dependency, the performance is also equivalent to MDSQ with a uniform central quantizer. Later, we show that MMDSQ achieves this performance but through a simpler manner without the dithering process.

2) Asymptotic Analysis of MMDSQ: Denote the rate of the first stage of each description as R_1 and that of the second stage of each description as R_2. Suppose that R_1 is high and the overload distortion is negligible. Since the two side quantizers are uniform, the side distortions by using such two side quantizers with entropy coding are [10]

$$D_{Q1} \approx \frac{2\pi e}{12} P_x 2^{-2R_1}. \tag{2}$$

When R_1 is sufficiently high, the side quantizer cell in the first stage is sufficiently small such that the pdf in each single cell is almost uniform. This gives $2^{2R_2} = N$, where N is the number of bins in the second stage quantizer. Since the joint quantizer reduces the first-stage bin by half, and the second stage divides the half-size bin into N uniform bins, the distortion is, thus, reduced by a factor of $(2N)^2$, and thus

$$D_{Q0} \approx \frac{1}{(2N)^2} D_{Q1} = \frac{2\pi e}{12}\frac{P_x}{4} 2^{-2(R_1+2R_2)}. \tag{3}$$

Now, observe that

$$\begin{aligned} D_{Q0}D_{Q1} &\approx \left(\frac{2\pi e}{12}\right)^2 \frac{P_x^2}{4} 2^{-2(2R_1+2R_2)} \\ &\approx \left(\frac{2\pi e}{12}\right)^2 \frac{P_x^2}{4} 2^{-4R} \end{aligned} \tag{4}$$

where $R = R_1 + R_2$ is the overall rate of each description. This is the same formula as given in (1), which shows that MMDSQ can achieve the same asymptotic performance as entropy-constrained MDSQ with a uniform central quantizer.

The factor $1/4$ in (4) comes from the staggering of side quantizers cells in the first stage. It is rather surprising that such a simple method enables MMDSQ to achieve almost all the performance that entropy-constrained MDSQ is able to achieve through complicated index assignments.

3) Relation Between MMDSQ and MDSQ: MMDSQ can be understood as a special class in the general MDSQ framework. This is most obvious when the second stage has $N = K^2$ number of bins, where K is also an integer. The first stage of MMDSQ is equivalent to a staggered index assignment, as shown in Fig. 3(a). The staggered index assignment previously appeared in [4] and [5] but was only used as a special index assignment pattern. The second stage is, in effect, an MDSQ using a full matrix as the index assignment matrix. Combining them together as in Fig. 3(b) results in an index assignment that is equivalent to the one given in Fig. 3(c). In this sense, MMDSQ uses this special class of index assignments, with the side quantizer choosing to ignore some of the available information: If an MDSQ adopts such an index assignment, the side quantizers will utilize the finer structure [Fig. 3(b)], while the side quantizers in MMDSQ ignore the fine structure from the second stage all together. It is surprising that without considering the details from the second (finer) stage, only the first stage suffices to provide a good asymptotic performance.

We emphasize the difference between MMDSQ and MDSQ. The most fundamental difference is that a side quantizer cell in MDSQ is not an interval but rather the union of several noncontiguous intervals; in contrast, a side quantizer cell in MMDSQ is always an interval. Second, the heuristic index assignment [2] for MDSQ will never result in an index assignment as the one

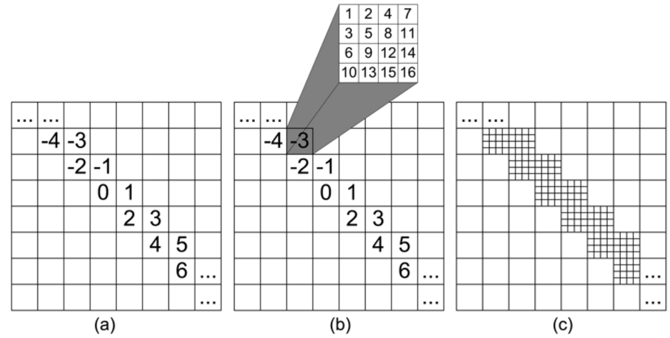

Fig. 3. (a) Staggered index assignment. (b) MMDSQ as two levels of MDSQs joined together. (c) Index assignment of the special class of MDSQ equivalent to MMDSQ.

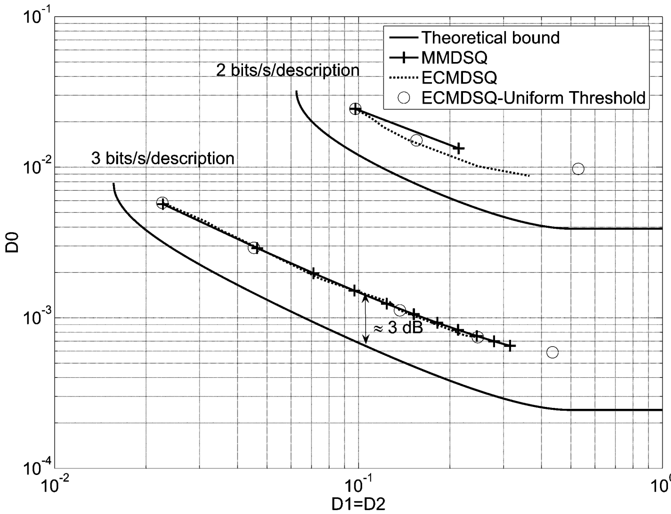

Fig. 4. Performance of MMDSQ at 3 bits/s/description and 2 bits/s/description. "+" marks the tradeoff points achieved by MMDSQ, while "o" marks the tradeoff points achieved by MDSQ with a uniform central quantizer.

in Fig. 3(c). Third, the tradeoff in MDSQ is controlled by the number of diagonals in the index assignment matrix: More diagonals in the index assignment matrix increase the side distortion but reduce the central distortion; in MMDSQ, the number of bins of the second stage quantizer controls the tradeoff: Having more bins in the second stage achieves the same effect as more diagonals in MDSQ. Most importantly, MMDSQ is not only simpler to implement but also provides a more flexible tradeoff control mechanism.

C. Implementation of MMDSQ

Center reconstruction is adopted here due to its simplicity, which is usually preferred in practical systems [5].

When the rate is low and the first stage cells are large, the N indices in the second stage are not equally probable, unlike the case at high rates; this fact can be utilized in performing the entropy coding.

To avoid problems in decoding variable-length entropy codes, the first stage information from a sample block are entropy coded to form the two bitstreams, and the second stage information of this block is split and appended to the two existing bitstreams. A special termination word can be added to signal the end of the first stage information, which results in a negligible overhead when the block length is large.

D. Numerical Results

Numerical methods are used to evaluate the performance of MMDSQ. The rate and distortions are computed with the source pdf and the corresponding (fictitious) codeword length, which provides the rate and distortion potentially attainable with ideal entropy coding. A unit-variance Gaussian i.i.d. source (with MSE distortion measure) is chosen, mainly because it is the only case for which the rate-distortion region is completely characterized [11]. The entropy-constrained MDSQ, as a reference system, is designed with the generalized Lloyd training algorithm in [3].

The performances of MMDSQ at different rates are shown in Fig. 4. At rates of 3 bits/s/description or above, MMDSQ achieves almost identical performance to that of entropy-constrained MDSQ, both of which are approximately 3.0 dB from the rate-distortion bound, as suggested by the analysis. The tradeoffs can be finely achieved, which are marked with "+" signs in the graph. In contrast, for entropy-constrained MDSQ to achieve such tradeoffs, extensive optimization of the codebook is required; when only uniform central quantizer cells are allowed, the available tradeoffs are very limited: For example, at 3 bits/s/description, only five tradeoff points can be achieved by using a different number of diagonals in the index assignment matrix over the same distortion range, which appear as circles in Fig. 4. At low rates, the performance of MMDSQ deteriorates, and the tradeoffs can only be achieved coarsely. This is not surprising because almost all uniform quantizers suffer from such an effect at low rates.

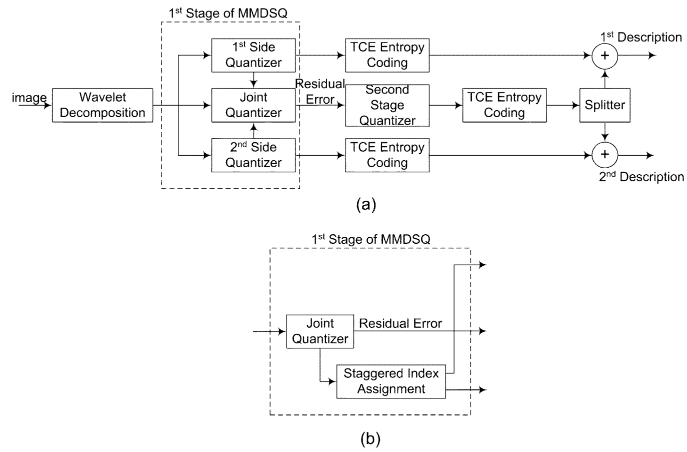

Fig. 5. (a) Structure of MMDSQ-TCE coder. (b) Alternative implementation of the first stage of MMDSQ.

III. Incorporating MMDSQ Into an Image Coder

A. Application of MMDSQ to the TCE Coder

We apply MMDSQ to a simple wavelet-based embedded image coder, which is the Tarp filter image coder with classification for embedding (TCE) [12]. This coder is selected for its simplicity and its good performance (which is comparable to JPEG-2000). It features bitplane coding, with the final quantizer having a deadzone that is twice the step size.

Applying MMDSQ to the TCE bitplane encoding is straightforward: The first stage's joint quantizer is uniform with step size Δ and a deadzone, and the second stage quantizes the residual. The entire encoding process is lossless, and lossy coding is achieved by truncating the bitstream at the desired rate in the second stage. More precisely, the coefficients are first quantized with the two staggered side quantizers; the indices from the two quantizers are then encoded by TCE independently to form the two side descriptions. The residual errors from the first stage are again encoded using TCE into a single bitstream, and this bitstream is split and appended to the two existing bitstreams [see Fig. 5(a)].

Notice that the first stage of MMDSQ can be implemented as the joint quantizer followed by a staggered index assignment block [see Fig. 5(b)], which is basically a division by 2 and can be efficiently implemented as a shift operation. The indices in the first stage retain most of the statistical structure of the coefficients, since the operation is almost linear; in contrast, the index assignment in MDSQ is a more nonlinear operation. To produce balanced descriptions, the symmetry in the probability distribution should be exploited, as described in [2]. The encoding of the residual errors in the second stage can use the information from the first stage; such information may be available only to the decoder when the first stage of both descriptions are present, which is always the case when the second stage will be decoded. This information includes *the significance map*, which indicates whether a coefficient has already become significant in the previous bitplanes.

B. Simulation Results

To provide a fair comparison, MDSQ is also applied to the TCE coder. We follow the basic algorithm in [5] (one of the first MD image coders but still providing competitive performance); however, the single-description image coder in [5] is replaced by the TCE coder. It was observed in [5] that to eliminate the necessity of training on the reconstruction points of the side quantizers, a side quantizer cell can choose the center of the central quantizer cell, which is within this side quantizer cell but is the closest to the origin, as its reconstruction point; this causes some performance loss. In contrast, it is not necessary for an MMDSQ-based system to make such a tradeoff between performance and implementation. It was also observed [5] that optimization of the number of diagonals in the index assignment matrix for each subband provides only minor improvement; thus, in the test coder, the number of diagonals in the index assignment matrix is kept constant for all the subbands when MDSQ is used. Similarly for MMDSQ, we keep the number of cells in the second stage constant for all the subbands.

A five-level wavelet decomposition is used with the Daubechies 9/7 filters, and all the subbands use the same quantizer. The test image is a 512-by-512 gray-scale *Lena* image. The performance of the original MDSQ-based coder in [5] is also included in the comparison as a reference, and it is denoted as "MDSQ+Wavelet" in Fig. 6, where the performance of three coders are provided. MMDSQ-TCE achieves the best performance among the three: MDSQ-TCE provides a performance inferior to MMDSQ-TCE, and they both achieve better performance than the state-of-the-art two-description image coder of [5]. Furthermore, MMDSQ-TCE easily achieves more tradeoff points than MDSQ-TCE.

The efficiency of MMDSQ-TCE is partially due to the efficiency of the TCE algorithm. However, such efficiency is not easily retained in the adaptation to MD, as suggested in [5], which is also partially confirmed by the difference between MMDSQ-TCE and MDSQ-TCE. By using MMDSQ, we are able to adapt an efficient image coder naturally to an MD system and retain its efficiency. Also note that though the analysis of MMDSQ is only for high rates, in this example, the rate 0.5 bits/pixel/description is not very high. When the rate is further reduced, MMDSQ-TCE keeps outperforming MSDQ-TCE, however, with a slowly diminishing performance gap.

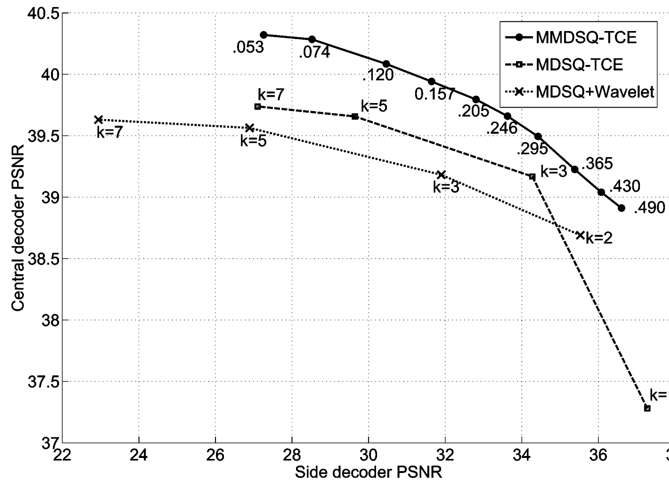

Fig. 6. Application of MMDSQ to image coding: results at 0.5 bits/pixel/description in PSNR. k is the number of diagonals used in the existing MDSQ, and for MMDSQ-TCE, each tradeoff point is labeled with the first stage rate per description R_1.

IV. Conclusion

We have proposed a new class of multiple description scalar quantizers, which features simple implementation and asymptotically good performance. The performance of this class of quantizers was evaluated both numerically and in an image coder. Because of the simplicity, such quantizers are suitable for practical multiple description systems, even when the rate budget is reasonably low. We provided the result of MMDSQ on image coding, and it confirmed the satisfactory performance of MMDSQ.

References

[1] V. K. Goyal, "Multiple description coding: Compression meets the network," *IEEE Signal Process. Mag.*, vol. 47, no. 6, pp. 2199–2224, Sep. 2001.

[2] V. A. Vaishampayan, "Design of multiple description scalar quantizers," *IEEE Trans. Inf. Theory*, vol. 39, no. 3, pp. 821–834, May 1993.

[3] V. A. Vaishampayan and J. Domaszewicz, "Design of entropy-constrained multiple description scalar quantizers," *IEEE Trans. Inf. Theory*, vol. 40, no. 1, pp. 245–250, Jan. 1994.

[4] C. Tian and S. S. Hemami, "Universal multiple description scalar quantizer: Analysis and design," *IEEE Trans. Inf. Theory*, vol. 37, no. 2, pp. 269–275, Sep. 2004.

[5] S. D. Servetto, K. Ramchandran, V. A. Vaishampayan, and K. Nahrstedt, "Multiple description wavelet based image coding," *IEEE Trans. Image Process.*, vol. 9, no. 5, pp. 813–826, May 2000.

[6] R. Zamir, "Gaussian codes and Shannon bounds for multiple descriptions," *IEEE Trans. Inf. Theory*, vol. 45, no. 7, pp. 2629–2636, Nov. 1999.

[7] V. A. Vaishampayan and J. C. Batllo, "Asymptotic analysis of multiple description quantizers," *IEEE Trans. Inf. Theory*, vol. 44, no. 1, pp. 278–284, Jan. 1998.

[8] A. El Gamal and T. M. Cover, "Achievable rates for multiple descriptions," *IEEE Trans. Inf. Theory*, vol. IT-28, no. 6, pp. 851–857, Nov. 1982.

[9] Y. Frank-Dayan and R. Zamir, "Dithered lattice-based quantizers for multiple descriptions," *IEEE Trans. Inf. Theory*, vol. 48, no. 1, pp. 192–204, Jan. 2002.

[10] R. M. Gray and D. L. Neuhoff, "Quantization," *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2325–2383, Oct. 1998.

[11] L. Ozarow, "On a source-coding problem with two channels and three receivers," *Bell Syst. Tech. J.*, vol. 59, pp. 1909–1921, Dec. 1980.

[12] C. Tian and S. S. Hemami, "An embedded image coding system based on tarp filter with classification," in *Proc. Int. Conf. Acoust., Speech, Signal Process.*, Montreal, QC, Canada, May 2004.